The Electric Potential Due to a Group of Charged Disks

1 Purpose

To find, using a 'virtual' application, the equipotential lines due to a complicated array of small disks maintained at constant voltage. This 'virtual' equipotential line map will be digitized and using a drawing software will be compared to a map generated by a computer software, such as, Desmos. The computer-generated map will be modeled using a 2-D point charge algorithm for the electric potential. The electric field stream lines of your 'virtual' equipotential line map will also be generated using the drawing software.

2 Theory

Typically, experimentally, small disks of silver or copper painted on semi-conductive paper are used to simulate point charges, but these will be simulated virtually using a Mathematica software provided by your professor. The 'virtual' field-mapping board of the application will be used to map the equipotential lines of various charge configurations. The electric field lines are then generated using the 'virtual' equipotential lines.

Applying Gauss' Law to a point charge q in 3-D space,

$$\epsilon_o \oint \vec{E} \cdot d\vec{A} = q_{enc},$$

we obtain

$$\epsilon_o E(4\pi r^2) = q$$

$$E_{3D} = \frac{q}{4\pi\epsilon_o r^2}.$$

The electric potential is obtained by integrating the electric field as

$$\Delta V = -\int \vec{E} \cdot d\vec{s}.$$

For the 3-D electric field around a point charge, the electric potential is given as

$$V = -\int E dr = -\frac{q}{4\pi\epsilon_o} \int \frac{dr}{r^2}$$

$$V_{3D} = \frac{q}{4\pi\epsilon_o r}$$

Applying Gauss' Law to a point charge in 2-D space, i.e. to a silver disk on a flat conducting sheet of paper,

$$\epsilon_o \oint \vec{E} \cdot d\vec{C} = q_{enc}$$

$$\epsilon_o E(2\pi r) = q$$

$$E_{2D} = \frac{q}{2\pi\epsilon_o r}$$

Integrating the electric field to obtain the electric potential, we get

$$V = -\int E dr = -\frac{q}{2\pi\epsilon_o}\int \frac{dr}{r}$$

$$V_{2D} = \frac{q}{2\pi\epsilon_o}\ln\frac{1}{r}$$

The principle of superposition is used calculate the net electric potential due to group of point charges,

$$V_{3D} = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} \frac{1}{4\pi\epsilon_o}\frac{q_i}{r_i}$$

$$V_{2D} = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} \frac{q_i}{2\pi\epsilon_o}\ln\frac{1}{r_i}$$

An equipotential line (2-D) or equipotential surface (3-D) is a line or surface which consists of all points that are at the same electric potential. No work is required to move a charge from one point to another on an equipotential line, and according to the definition of work

$$W = \int_a^b q\vec{E}\cdot d\vec{l} = 0,$$

this can only be satisfied if the electric field is ALWAYS perpendicular to the equipotential lines. Thus by mapping out the equipotential lines one can generate the electric field lines.

3 Procedure

This 'virtual' experiment is divided into three parts: one part is finding and recording equipotential points around six (6) disk-shaped point charges on a 'virtual' semi-conducting sheet. In an 'interpretation of results' these equipotential points are digitized to a drawing software to generate the quipotential lines. The second part is using the Desmos online computer software to calculate, using the 2-D electric potential function, the electric potential on a grid of points to generate a map of equipotential lines for your particular charge configuration, and the third part, as an 'interpretation of results', is to obtain the electric field stream lines for your experimental equipotential lines.

3.1 Virtual Experiment

1. Using the Exp6.studentversion.2026.cdf file and Wolfram Player software you will choose your point charge configuration based on your section and group number. On a blank white copy of the conductive paper template draw in your six (6) painted disk as found on your 'virtual' conductive paper. Use a "dime" to draw in the circles representing the point charges. The red charges are at an electric potential of +5V and the black charges are at a potential of -5V. Mark your drawn disks with the +5V or -5V.

2. Record the coordinate and effective charge of each disk, see row of numbers at the bottom of the virtual page.

3. Use the mouse to move the black dot locator on the 'virtual' paper to obtain the electric potential at a given coordinate to find five equipotential lines
$V = -3, -1.5, 0.0, 1.5,$ and $3.0\,V$ and map out the equipotential points on your blank copy of the conductive paper template. Around the positive charge you should find a +3V and perhaps a +1.5V line, around the negative charge you should find a -3V and perhaps a -1.5V line. In between positive and negative charges you should find a 0V line. My suggestion is to give your partner the y-coordinate at each x-line of your target electric potential as you move the probe horizontally and give the x-coordinate at each y-line of your target electric potential as you moved the probe vertically using the snap button and have your partner mark a point at that coordinate. Make sure to label that set of points with an electric potential. You are to follow the equipotential lines to the edge of the 'virtual' paper. Find a sufficient amount of points, not more than a spacing between two points of one unit, to clearly define the equipotential line.

4. Take a screenshot of your 'virtual' conductive paper with your disks using the snipping tool.

3.2 Calculating the equipotential lines

1. Use the Desmos template at

   `https://www.desmos.com/calculator/abhghctbe2`

   as a starting point to calculate the voltage (V_{2D}) for your 28 x 20 grid. Change the center coordinates of each disks to correspond to your charge configuration. Black corresponds to -5V and red to +5V. Change the zeroes in the distance equations with the x and y coordinate of each disk. And then change the effective charge to the ones given in the bottom row of the 'virtual' paper, where the effective charge is the third term of (x1, y1, q1) coordinate. If done correctly you should have generated equipotential lines for your group charge configuration. Save and give a name to your session. Using the snipping take a screenshot of the webpage including the web URL direction and a screenshot of your equipotential map.

2. At this point, you should have a white copy of the 'virtual' conductive paper template with your charges drawn in, with the equipotential points of -3V, -1.5V, 0, 1.5V, and 3V, and the the equipotential line plot generated by Desmos.

4 Interpretation of Results

1. Scan your white paper template with the generated equipotential points using your smartphone (the camera as close as possible), a scanning app is useful to straighten out the picture. And, transfer the picture to the computer and open the Inkscape software. Copy and paste the scanned picture of your white paper template with the generated equipotential points in the first layer of a new Inkscape project. Resize it so that it fits on the canvas page and lock this layer. Create and add a new layer (using the layer tab) and go to this second layer. In your lab report please describe what you have done in complete sentences and including a screenshot of your Inkscape window.

2. On this second layer you will draw red and black disks corresponding to your charges. In your lab report please describe what you have done in complete sentences and including a screenshot of your Inkscape window.

3. Make your first layer visible. Also on the second layer you will connect the equipotential points of the same voltage using the 'Draw Bezier curves and straight lines' icon to give a 'rough' equipotential line. You will select the curve nodes of the single equipotential lines using the 'Edit paths by nodes' icon, and click-on the 'Make selected nodes auto-smooth' button. Identify your equipotential lines -3V, -1.5V, 0V, 1.5V, and 3V and mark them using the Text tool. Include on this layer a rectangle that matches the outer frame of the grid of layer 1. Lock this second layer. With layer 1 and 2 visible make a copy of your work using the Snipping Tool and include it in your report. Please describe what you have done in complete sentences.

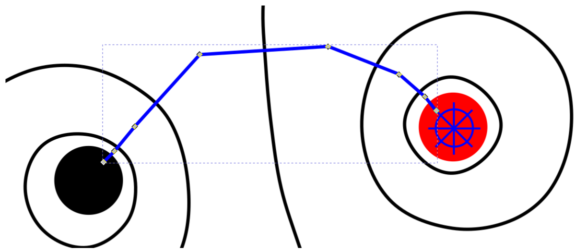
4. Explain how the equipotential lines were generated using the Desmos online software.

5. In this interpretation, you want to compare your calculated equipotential lines (from Desmos) to the virtual equipotential lines generated in interpretation 3. Create and add a new layer (3rd layer) to your Inkscape project. Layers 1 and 3 should be visible. Copy the 2-D Desmos equipotential lines plot, using the snipping tool, and paste this to the third layer. Reduce the opacity of the Desmos picture to make it transparent in order to see the plot of layer 1. Adjust the size of the picture of the Desmos plot such that the outer frame of the plot matches the grid of the layer-1 plot. Lock layer-3. Make layer-1 invisible and make layer-2 visible. You should be able to see the equipotential lines of both plots. With layers 2 and 3 visible make a copy of your work using the Snipping Tool and include it in your report. Please describe what you have done in complete sentences.

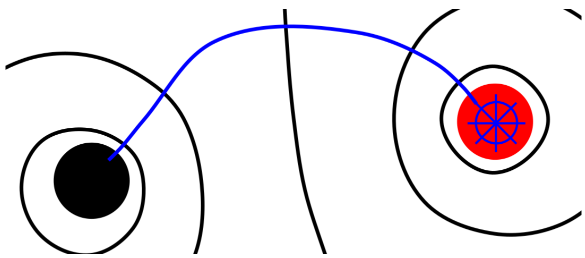
6. From a qualitative point of view, where on the plot are the equipotential lines similar and where are they different? Think edges of the map as opposed to middle of the paper. How are they different?

7. In this interpretation an electric field lines map will be created. Open your Inkscape project and create and add a 4th layer. You will insert a copy of the following inside each of the positive disk. You can reduce the opacity so that you could see the disks of layer 2, adjust the size such that the blue circle matches the disk.

   Using the 'Draw Bezier curves and straight lines' icon you will start a straight line on the ring of the above figure at one of the intersection points and move perpendicular out of the disk towards a point in between two equipotential lines and click the mouse, then continue moving making sure that the line crosses always perpendicular to an equipotential line, towards a negative charge. The following picture is a guide.

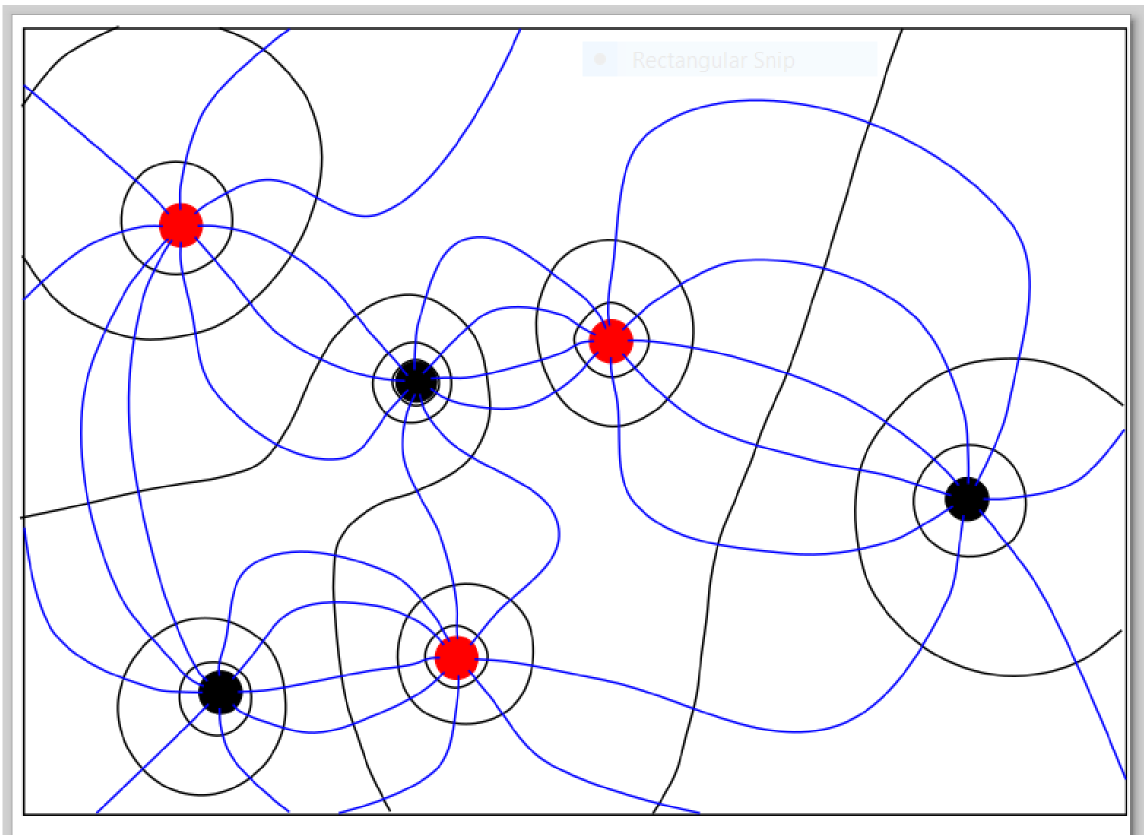


Some electric field lines will intersect the border of the template before reaching a negative charge. If you reach a negative charge terminate the field line in the outer part of the disk. Remember the disks are conductors and the surface forms an equipotential and the electric field lines should be perpendicular to the disk perimeter. Then, you will select the curve nodes of the electric field line using the 'Edit paths by nodes' icon, and click-on the 'Make selected nodes auto-smooth' button. This action should transform the segmented line to a smooth curve as illustrated in the following picture.



Draw at least 6-8 electric field lines per charge, the electric field cannot cross each other and are always perpendicular to the equipotential lines. Your final map should look similar to the following figure.

Layers 1 through 4 should be locked and layer-2 and 4 visible. With layers 2 and 4 visible make a copy of your work using the Snipping Tool and include it in your report. Please describe what you have done in complete sentences.


Rectangular Snip