A Thermodynamic Cycle

1 Purpose

Experimentally create a thermodynamic cycle and analyze all thermodynamic processes to determine the net work, net heat, and thermal efficiency.

2 Theory

Our thermodynamic system will consist of air composed of diatomic molecules in a cylinder. The volume of the cylinder can change due to a movable piston. Our system will undergo four (4) thermodynamic processes where the initial and final state of the system will be the same giving a closed thermodynamic cycle.

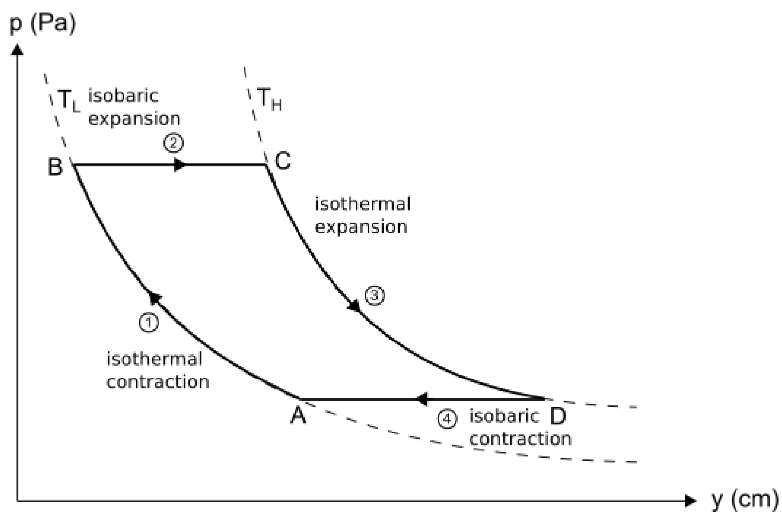

Figure 1: Our thermodynamic cycle

The initial state at A will be at T_L, p_A, no mass on piston, and piston position y_A. The volume V_A of our system consists of the volume of the heat reservoir (hr) plus the volume of the tubing (tu) plus the volume of the piston cylinder (pc) with piston at height y_A.

$$V_A = V_{hr} + V_{tu} + V_{pc} = A_{hr}\, h_{hr} + A_{tu}\, l_{tu} + A_{pc}\, y_A \tag{1}$$

where the cross-sectional areas are given by

$A_{hr} = \pi\,(r_{hr})^2$, $A_{tu} = \pi\,(r_{tu})^2$, and $A_{pc} = \pi\,(r_{pc})^2$.

The processes will consist of:

1. $A \to B$ an isothermal compression of the air where work is performed on the system and heat is removed from the system,
2. $B \to C$ an isobaric expansion of the air where heat is added to the system and work is performed by the system,
3. $C \to D$ an isothermal expansion of the air where work is performed by the system and heat is added to the system,
4. $D \to A$ an isobaric compression of the air where heat is removed to the system and work is performed on the system.

At this point the state of the system should have returned to the initial state A.

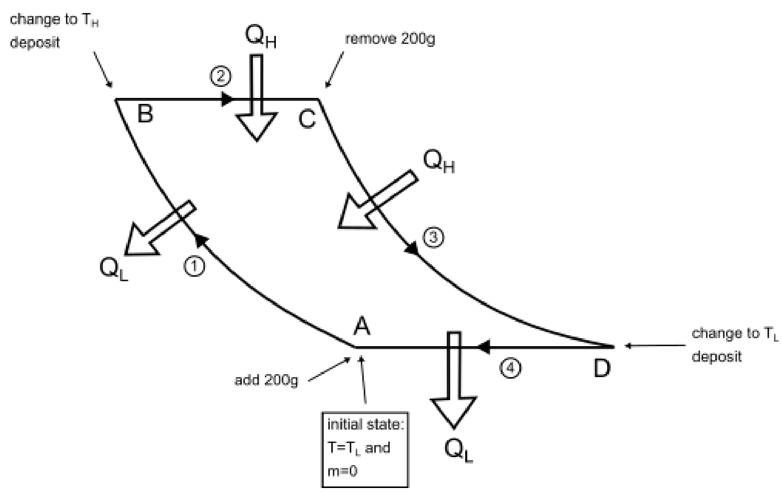

Figure 2: Heat and work in all processes

All thermodynamic processes of our cycle will follow the 1^{st} law of thermodynamics

$$Q = \Delta E_{int} + W \tag{2}$$

for the:

1. $A \to B$ process the 1^{st} law gives $Q_L = W$,
2. $B \to C$ process $Q_H = nC_p\Delta T$, $\Delta E_{int} = nC_V\Delta T$.
3. $C \to D$ process the 1^{st} law gives $Q_H = W$,
4. $D \to A$ process $Q_L = nC_p\Delta T$, $\Delta E_{int} = nC_V\Delta T$.

As air consists of mostly diatomic molecules, the molar heat capacities are $C_V = 5/2\,R$ and $C_p = 7/2\,R$, where $R = 8.3145\,\frac{\text{J}}{\text{mol K}}$ is the ideal gas constant.

We will make the assumption that the air behaves as an ideal gas such that

$$p\,V = n\,R\,T \tag{3}$$

3 Procedure

Determination of heat reservoir, tubing, and heat engine initial volumes

1. Measure the diameter and length of the heat reservoir canister using the vernier caliper in cm. The length is from the bottom of the aluminum canister to the top surface of the rubber stopper. Remove the rubber stopper and measure its height in cm. Re-assemble the heat reservoir.

2. Measure the inside diameter of the tubing using the vernier caliper in cm and estimate the length in cm of all tubing connected to the cylinder/piston using a string and ruler.

3. Fill-out the following table.

diameter heat reservoir (cm)	
length heat reservoir with stopper(cm)	
heat reservoir stopper height (cm)	
inside diameter tubing (cm)	
length tubing (cm)	
diameter piston (cm)	
initial height piston (cm)	

Table 1: Diameters, lengths, and heights of components

Assembly and verification

4. Assemble the heat engine apparatus (HEA) and rotary motion sensor (RMS) as shown by your instructor. The piston platform should be connected to a counterweight of mass equal to the piston/platform mass listed on the HEA. Lift the platform and hold it in place using the side screw at more or less at 40% height. Connect the cylinder/piston to a heat reservoir and absolute pressure gauge. Release temporarily the side screw and if there are no leaks, the piston should be stationary. Again tighten the side screw to hold the platform in place.

Observation a thermodynamic cycle

5. Prepare your hot and cold heat deposits. The cold heat deposit will consist of a mixture of ice and water at 0°C. The hot heat deposit will consist of water simmering at 100°C in the steam generator. Insert both thermometers in the cold heat deposit in order to calibrate the thermometers. Once calibrated move the thermometer without the masking tape to the hot heat deposit.

6. Place the heat reservoir in the cold heat deposit and periodically agitate the ice/water mixture. Remove all masses from the piston platform, disconnect the pressure sensor to open the system to the atmosphere and loosen the side screw and move the freely-moving piston to more or less 40% height (30

7. Start the Capstone software. The following four steps should be performed successively and relatively quickly.

   (a) add 200g to the platform and wait momentarily until the piston height has stabilized, then immediately

   (b) switch the heat reservoir to the high temperature deposit and momentarily wait until the piston height has stabilized, then immediately

   (c) remove the 200g from the platform and momentarily wait until the piston height has stabilized, then immediately

   (d) switch the heat reservoir to the low temperature deposit and momentarily wait until the piston height has stabilized and has reached the initial state and stop the Capstone software.

8. Using capstone, highlight the data points that are out of place and one can delete those highlighted data points, you basically want to clean-up the cycle such that it looks like the following:

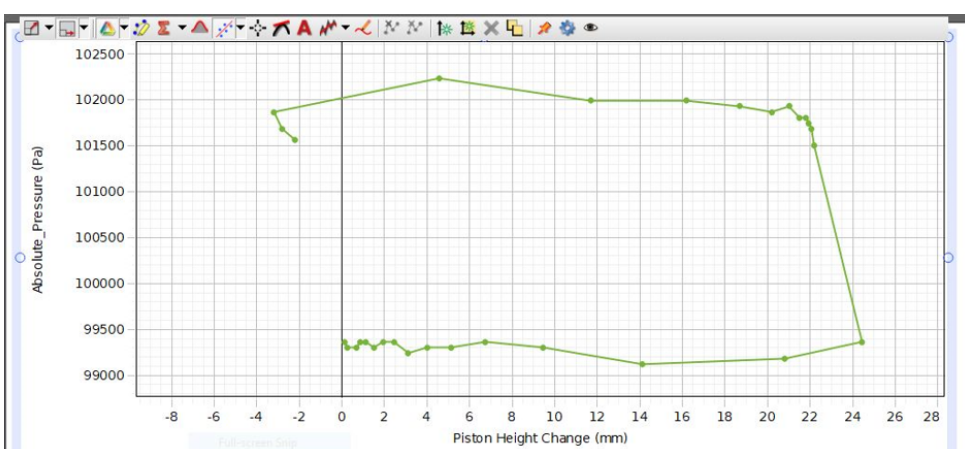

Figure 3: Cleaned up capstone cycle

9. Copy the pressure and piston position data of your thermodynamic cycle to Excel and save the Excel file. Copy and save a screencapture of your Capstone window.

4 Interpretation of Results

1. Copy the last point of your pressure vs piston data and paste it in cells at the top of your data so that it also becomes your first point.

2. Determine the initial volume of the system V_A in cm^3 using equation 1 and the cross-sectional areas A_{hr}, A_{tu}, and A_{pc} and the initial height of the piston y_A. Convert this initial volume to m^3 using the following conversion factor: $1.0\,cm^3 = 1.0\,cm^3 \times \left(\frac{1m}{100cm}\right)^3 = 1.0 \times 10^{-6}\,m^3$.

3. During the experiment, the piston moves, changing the volume of the system. Convert the change of piston height Δy (mm) of your Excel data to a volume using $V_i = V_A + A_{pc}\,\Delta y_i \times \left(\frac{1\,m}{1000\,mm}\right)$ where Δy_i is the change of the piston cylinder position at any point i in your thermodynamic cycle in mm as recorded in Capstone and where A_{pc} **is converted to** m^2, be careful with the units, the volumes in m^3 shoul be in the order of $10^{-4}\,m^3$. Plot your thermodynamic cycle: absolute pressure $p\,(Pa)$ versus system volume $V\,(m^3)$.

4. Using your data and graph of the above interpretation, determine the volume and pressure, i.e. the coordinates (V,p) of the points that correspond to the points A, B, C, and D of your thermodynamic cycle. These points are essentially the vertices of your thermodynamic cycle.

5. If there are no air leaks in the system, the number of moles n of air during a cycle does not change such that using the ideal gas law gives

$$n = \frac{p_A\,V_A}{R\,T_A} = \frac{p_B\,V_B}{R\,T_B} = \frac{p_C\,V_C}{R\,T_C} = \frac{p_D\,V_D}{R\,T_D} \tag{4}$$

   However, the system will most likely leak between points $A \to B \to C$ when the 200g is added to the platform so the number of moles will change during the cycle, therefore an average number of moles will be calculated. Calculate the number of moles n of air at points A, B, C, and D of your thermodynamic cycle and average the results. Recall: use the absolute temperature scale in the ideal gas law.

6. Calculate the heat added to the system during the thermodynamic cycle $Q_H = Q_{B\to C} + Q_{C\to D}$ using the following analysis.

   (a) Calculate $Q_{B\to C}$.

   For the $B - C$ isobar, $Q_{B\to C} = n\,C_p\,(T_C - T_B)$ where C_p is the diatomic gas molar heat capacity at constant pressure and is equal to $C_p = 7/2\,R$. $T_C - T_B$ is the temperature difference between the high and low temperature heat deposit. So then

$$Q_{B\rightarrow C} = 7/2\, nR\, (T_H - T_L) \tag{5}$$

(b) Calculate $Q_{C\rightarrow D}$.

For the $C - D$ isotherm,

$$Q_{C\rightarrow D} = nR\, T_D\, \ln \frac{V_D}{V_C} \tag{6}$$

(c) Calculate $Q_H = Q_{B\rightarrow C} + Q_{C\rightarrow D}$.

7. Calculate the heat removed from the system during the thermodynamic cycle $Q_L = Q_{D\rightarrow A} + Q_{A\rightarrow B}$ using the following analysis.

   (a) Calculate $Q_{D\rightarrow A}$.

   For the $D - A$ isobar, $Q_{D\rightarrow A} = n\, C_p\, (T_A - T_D)$ where C_p is the diatomic gas molar heat capacity at constant pressure and is equal to $C_p = 7/2\, R$. $T_A - T_D$ is the temperature difference between the low and high temperature heat deposit. So then

$$Q_{D\rightarrow A} = 7/2\, nR\, (T_L - T_H) \tag{7}$$

   (b) Calculate $Q_{A\rightarrow B}$.

   For the $A - B$ isotherm,

$$Q_{A\rightarrow B} = nR\, T_B\, \ln \frac{V_B}{V_A} \tag{8}$$

   (c) Calculate $Q_L = Q_{D\rightarrow A} + Q_{A\rightarrow B}$.

8. Determine the net work done during the thermodynamic cycle. As a first approximation of the net work or area enclosed by the thermodynanic cycle, the area enclosed by the quadrilateral defined by the four (V,p) coordinates points A, B, C, and D will be calculated using Heron's formula for the area enclosed in an arbitrary quadrilateral. A quadrilateral consists of two triangles.

$$A_{tri} = \sqrt{s(s-a)(s-b)(s-c)}$$

   where a, b, and c are the lengths of the sides of the triangle, and $s = \frac{a+b+c}{2}$.

   Use the Excel sheet provided by your professor to calculate the enclosed area of your quadrilateral formed by your points A, B, C, and D.

9. In Excel, numerically find the area under each process of the cycle to calculate the work done for each process: A to B, B to C, C to D, and D to A. Split your pressure and volume data according to the four processes, and find the area numerically under the curve for each process. Recall that the work should be positive during expansion of the gas and negative during contraction of the gas. The net work is given by

$$W_{net} = W_{A\to B} + W_{B\to C} + W_{C\to D} + W_{D\to A} \tag{9}$$

What is the net work for your thermodynamic cycle? It should be similar to the enclosed area of your quadrilateral. Explain why the results are not identical.

10. How does the net work obtained in the previous interpretation compare to the mechanical work in lifting up 200g from position B (lowest) to position C (highest)? Obtain the lowest position at point B and the final position at point C in the original Capstone data in Excel. We don't consider position C to D as there is no mass on the piston during that process. Recall: $W = mg\,\Delta y$. Make sure to convert all quantities to SI units, kg, m, and s. Compare the thermodynamic and mechanical work using a percentage difference.

11. From the law of conservation of energy applied to a thermodynamic cycle, $W_{net} = |Q_H| - |Q_L|$, calculate W_{net} using the results of interpretations 6 and 7, and compare to the net work calculated obtained in interpretation 9 using the percentage difference.

12. Determine the efficiency ϵ of your thermodynamic cycle using the W_{net} from interpretation 9 and

$$\epsilon = \frac{W_{net}}{Q_H} \times 100\% \tag{10}$$

and compare to the ideal efficiency (of a Carnot cycle)

$$\epsilon_{Ct} = \left(1 - \frac{T_L}{T_H}\right) \times 100\% \tag{11}$$