Impulse, momentum, and elastic collisions

1 Purpose

To verify that impulse is equal to the change of momentum. The impulse will be calculated by integrating various force versus time graphs using the data studio software. We will compare these results to the change of momentum as determined from the experimental data. In addition the magnetic force between two magnets will be analysed. We will also verify that the collison between two carts with magnetic repulsion is elastic.

2 Theory

According to Newton's Second Law,

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \rightarrow \vec{F}_{net}dt = d\vec{p} \tag{1}$$

integrating

$$\int \vec{F}_{net}dt = \int d\vec{p} \rightarrow \vec{J} = \Delta\vec{p} \tag{2}$$

$$\text{impulse } \vec{J} = \int \vec{F}_{net}dt \tag{3}$$

In the above equation the IMPULSE is defined. Due to the impulse on an object we observe a change in momentum of the object. Impulse is obtained by integrating or finding the AREA under a force versus time curve.

The impulse due to a constant average force (of a changing force) can be obtained using

$$J = \int F_{net}dt = F_{av}t_c \tag{4}$$

where t_c is the time of contact between two objects.

In this experiment the net force on a moving object will be the repulsive magnetic force between two similar magnetic poles.

During elastic collisions between two bodies in an isolated system the total momentum and kinetic energy are conserved. This gives the following relations

$$v_2{}' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_2 \tag{5}$$

$$v_1{}' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 + \left(\frac{2m_2}{m_1 + m_2}\right) v_2 \tag{6}$$

if $m_1 = m_2$, then the two above equations simplify to:

$$v_2{}' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1 \rightarrow v_2{}' = v_1 \tag{7}$$

$$v_1{}' = \left(\frac{2m_2}{m_1 + m_2}\right) v_2 \rightarrow v_1{}' = v_2 \tag{8}$$

3 Procedure

1. Assemble the equipment as instructed.
2. Measure the mass of the two carts. Add mass to one cart such that both have equal mass.
3. Level horizontally the end of the linear airtrack where the force sensor is mounted.
4. Position one of the two photogates as close as possible to the force sensor where the repulsive magnetic force on the cart is zero. Measure the distance between this photogate and the magnet of the force sensor. The other photogate should be approximately 35-45 cm further.
5. You will propel the left-hand cart at different velocities towards the right using an electromagnet at different voltages while temporarily holding the right-hand cart between the two photogates. Due to the collision the right-hand cart will move to the right towards the force sensor. Make sure the force sensor is tared just before the collision of the two carts.
6. Obtain the initial velocity of the left-hand cart and the initial and final velocities of the right-hand cart, area under a force vs. time curve, maximum force, and closest separation for 10 different trials. Use your smartphone video capability or bamboo stick to obtain the closest separation.

7. Your data can be recorded in a table such as following:

cart mass (g)			photogate force sensor magnet distance (cm)					
cart magnet thickness (cm)				force sensor magnet position (cm)				
Run	Voltage (V)	$\vec{v}_1$ (m/s)	$\vec{v}_2'$ (m/s)	$\vec{v}_2'$ (m/s)	$\vec{v}_2''$ (m/s)	area (Ns)	$F_{max}(N)$	x_f (cm)

8. Screen capture one of your datastudio windows showing the bell-shaped force vs. time curve with area info and velocity tables.

4 Interpretation of Results

1. Plot a graph of the final velocity (after the first collision) of the right-hand cart versus the initial velocity of the left-hand cart. Use linear regression obtain the best-fit slope and compare it (using % error) to the theoretical value of 1, since $v_2{}' = 1\ v_1$. Do the results of this experiment demonstrate that the collison between two carts with magnetic repulsion is elastic?

2. Calculate the magnitude of the change in momentum Δp for each run using the mass of the right-hand cart and initial and final velocities (in the second collision with the end of the linear airtrack) of the right-hand cart. Remember that data studio does not recognize directions and that a negative sign must be assigned to one of the velocities.

3. Plot a graph of the magnitude of the impulse $\vec{J}$ (area under a F vs t curve) vs. the magnitude of $\Delta\vec{p}$. Use a linear regression to obtain the best-fit slope and compare it (using % error) to the theoretical value of 1, since $J = 1\ \Delta p$. Do the results of this experiment demonstrate the validity of the relationship between impulse and change of momentum?

4. Estimate the average force exerted between the magnets for your force vs time screen capture.

5. Plot a graph of maximum force versus closest separation distance in meters. Does it show an inversely proportional to separation behavior? How can you tell?

6. Suppose that the magnetic force between two similar magnetic poles has the form of: $F(r) = \frac{k}{r^n}$, a Log-Log plot will linearize the data since $\ln F = -n \ln r + \ln k$. Plot a graph of ln(F) vs. ln(r) where r is the nearest separation (in meters) between the magnetic poles of each magnet and F is the maximum force. Perform a linear regression of the data to obtain the exponent n of the separation r and y-intercept. The magnetic force between two magnets is complex and depends on the geometry of the magnets and can give a non-integer exponent. With two small dipole magnets the interaction should be inversely proportional to r^4. Compare your experimental determination of the exponent to the theoretical exponent $n = 4$ using the % difference. Use the y-intercept to find k and determine the functional form of the magnetic force $F(r)$ between two magnets, for example such as $F(r) = \frac{0.000257}{r^{3.167}}$.

7. Plot a graph of $F(r)$ vs. r using scatter with continuous lines in Excel and superimpose the data points of the graph of interpretation 5. How does the F(r) curve correlate with the data points?

8. Use your $F(r)$ of the previous interpretations to estimate the magnetic force on the right-hand cart as it passes the right-most photogate. Is the magnetic force at that distance negligible?