Moment of Inertia

1 Purpose

To determine experimentally and theoretically the moments of inertia of a disk and rings of definite thickness about various rotational axes. To show that the moment of inertia depends on the geometry of the rotating object, the moment of inertia of objects sharing an axis of rotation are additive, that the moment of inertia increases as the distribution of mass moves farther away from the axis of rotation, and to study the relationships between torque and rotational motion.

2 Theory

Newton's second law for rotation is given as

$$\sum \tau = I\alpha \tag{1}$$

where τ are the torques on the rotating object, I is the moment of inertia of the rotating object, and α is the angular acceleration of the rotating object.

The moment of inertia I plays essentially the same role in rotation as the mass m does in translational motion. The moment of inertia is the property of a body to resist changes of its rotational state about a rotational axis.

2.1 Review of rotational motion

Rotation Equations	Relation to Linear Motion
$\omega = d\theta/dt$	$v = \omega r$
$\alpha = d\omega/dt = d^2\theta/dt^2$	$a = \alpha r$
$\tau = r_\perp F$	
$\tau = I\alpha$	$F = ma$

From the diagram below we see that the applied torque τ_a on the rotating platform is due to the tension T of the string that is wrapped around the axis of rotation.

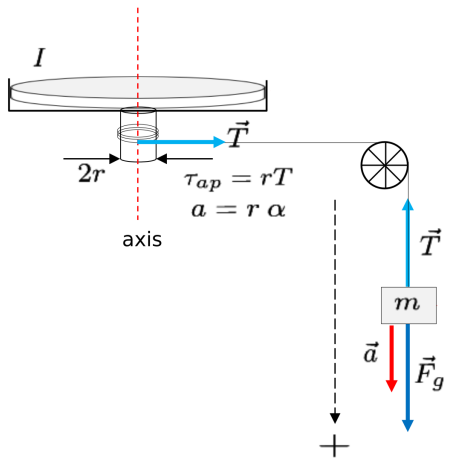


Figure 1

The net torque τ_{net} on the rotating platform is given by

$$\tau_{net} = \tau_a - \tau_f \tag{2}$$

where τ_f is a frictional torque. The net torque of the rotating platform is also equal to

$$\tau_{net} = I\alpha. \tag{3}$$

Manipulating the above two equations gives

$$\tau_a = I\alpha + \tau_f \tag{4}$$

The applied torque τ_a can be calculated as

$$\tau_a = rT \tag{5}$$

and the angular acceleration α can be obtained from the linear acceleration as

$$\alpha = a/r \tag{6}$$

Using Newton's second law, the net force on the hanging weight is

$$F_{net} = mg - T = ma$$

$$T = mg - ma \tag{7}$$

such that the tension T can also be calculated using the acceleration.

The moment of inertia I is actually the sum of all of the moments of inertia of each component bodies, i.e.

$$I = I_s + I_a \tag{8}$$

where I_s is the moment of inertia of the rotating support and I_a is the moment of inertia of the accesory body.

The moment of inertia of a disk and rings with respect to different axes orientation are given in the following figure.

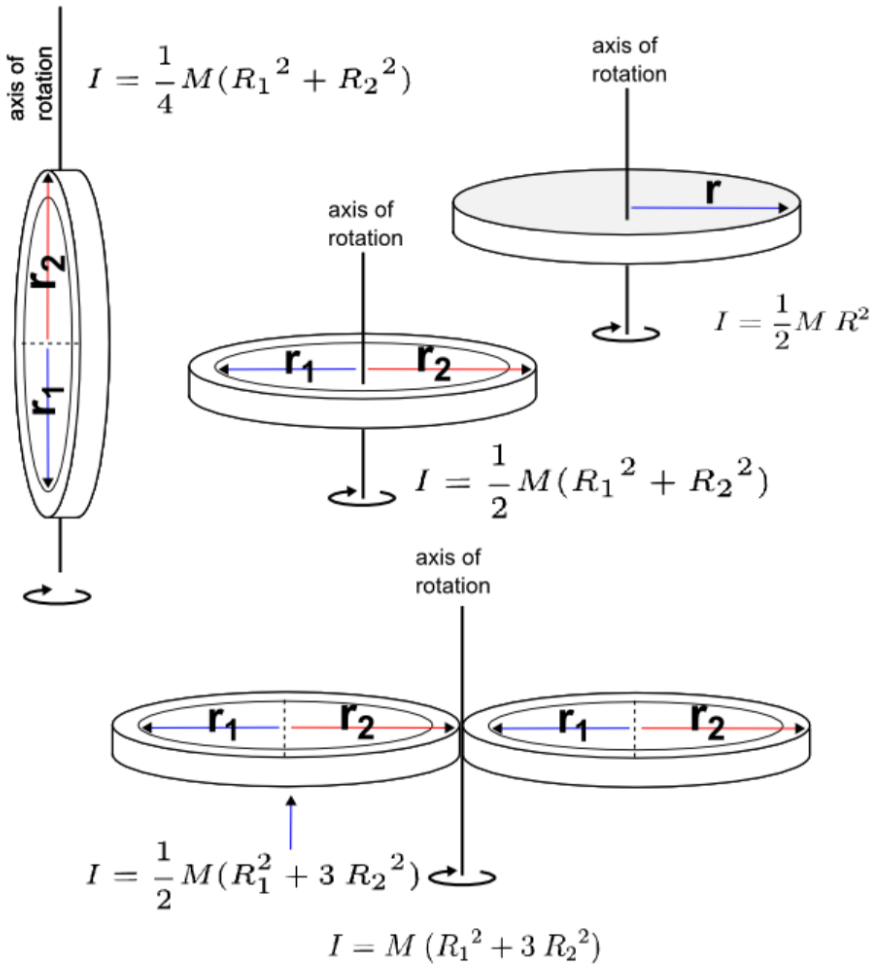


Figure 2

3 Procedure

1. Assemble equipment as directed by your lab instructor. The long bar will be mounted on the rotary support using two screws.

2. Measure:

- hanger mass
- the diameter of the rotary support axis
- the mass and diameter of the disk
- the diameter of the inner and outer edge of both cylindrical rings
- the mass of both cylindrical rings

hanger (g)	
diameter rotary support (cm)	
mass disk (g)	
diameter disk (cm)	
mass ring 1 (g)	
outer diameter ring 1 (cm)	
inner diameter ring 1 (cm)	
mass ring 2 (g)	
outer diameter ring 2 (cm)	
inner diameter ring 2 (cm)	

3. Measure the acceleration of 5 different hanging weights, hanger ($\approx$ 50g) + up to 6 x 20g weights, using the smart pulley arrangement for the following configurations:

   - rotary support with attached long bar (3a) [hanger + 4 x 20g]

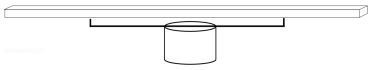


Figure 3a

   - rotary support with attached long bar with **single disk** (3b) mounted horizontally [(hanger + 20g) + 4 x 20g]

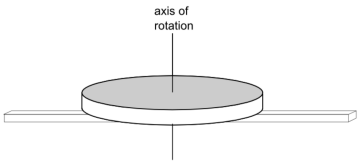


Figure 3b

- rotary support with attached long bar with **single ring** (3c) mounted horizontally [(hanger + 20g) + 4 x 20g]. Note which of the two rings you are using.

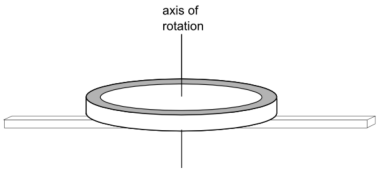


Figure 3c

- rotary support with attached long bar with **two rings** (3d) mounted horizontally one on top of the other [(hanger + 40g) + 4 x 20g]

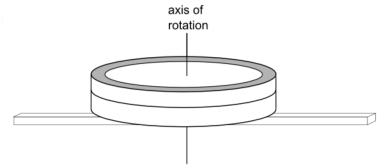


Figure 3d

- rotary support with attached long bar with **two rings** (3e) mounted horizontally side by side [(hanger + 40g) + 4 x 20g]

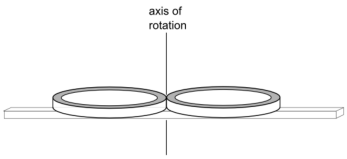


Figure 3e

4. Arrange your data as such:

To measure			To calculate		
Configuration	hanging mass m (g)	a (m/s^2)	T (N)	τ_a (Nm)	α (rad/s^2)
support+bar Fig. 3a					
support+bar Fig. 3a					
support+bar Fig. 3a					
support+bar Fig. 3a					
support+bar Fig. 3a					
support+bar + 1 horizontal disk Fig. 3b					
support+bar + 1 horizontal disk Fig. 3b					
support+bar + 1 horizontal disk Fig. 3b					
support+bar + 1 horizontal disk Fig. 3b					
support+bar + 1 horizontal disk Fig. 3b					
support+bar + 1 horizontal ring Fig. 3c					
support+bar + 1 horizontal ring Fig. 3c					
support+bar + 1 horizontal ring Fig. 3c					
support+bar + 1 horizontal ring Fig. 3c					
support+bar + 1 horizontal ring Fig. 3c					
support+bar + 2 horizontal rings Fig. 3d					
support+bar + 2 horizontal rings Fig. 3d					
support+bar + 2 horizontal rings Fig. 3d					
support+bar + 2 horizontal rings Fig. 3d					
support+bar + 2 horizontal rings Fig. 3d					
support+bar + 2 horizontal rings Fig. 3e					
support+bar + 2 horizontal rings Fig. 3e					
support+bar + 2 horizontal rings Fig. 3e					
support+bar + 2 horizontal rings Fig. 3e					
support+bar + 2 horizontal rings Fig. 3e					

4 Interpretation of Results

1. For each configuration, plot τ_a vs. α . Use linear regression to find the best-fit slope. The slope of your linear regression gives the TOTAL moment of inertia. Do a format trendline label to increase the precision of the slope to 4 significant figures. Summarize your total moments of inertia and configurations in a table.

2. The analysis of this interpretation should show that by changing the geometry (and mass distribution) of the rotating object with same axis of rotation, the moment of inertia will also change.

   The total moment of inertia for the support+bar + disk configuration is $I_{tot} = I_{s+b} + I_{disk}$. Use $I_{disk} = I_{tot} - I_{s+b}$ and your results of interpretation 1 to determine the moment of inertia of the disk of Fig. 3b. Use $I_{ring} = I_{tot} - I_{s+b}$ and your results of interpretation 1 to determine the moment of inertia of the ring of Fig. 3c.

   Determine the ratio of the experimental moment of inertia of the ring to the experimental moment of inertia of the disk, $\frac{I(exp)_{ring}}{I(exp)_{disk}}$. The mass of the disk is uniformly distributed from the center to the edge giving a smaller moment of inertia to that of a ring of similar mass and diameter. Theoretically, using the equations in Figure 2, the ratio of $\frac{I_{ring}}{I_{disk}}$ should equal to $\frac{m_{ring}(r_1^2+r_2^2)}{m_{disk}r^2}$. Calculate $\frac{m_{ring}(r_1^2+r_2^2)}{m_{disk}r^2}$ and compare using the % difference the experimental ratio of $\frac{I(exp)_{ring}}{I(exp)_{disk}}$ to the theoretical ratio of $\frac{m_{ring}(r_1^2+r_2^2)}{m_{disk}r^2}$, use the mass and radii of the disk and ring that you have used in Figures 3 b and c.

   Use the equations for the moments of inertia of Figure 2 to determine the theoretical moments of inertia of the Figure 3b (disk) and 3c (ring) configurations. Use the mass and radii of the disk and ring that you have used in Figures 3 b and c. Compare these theoretical results with the correponding experimental moments of inertia previously determined in this interpretation using percentage difference. Generate a table summarizing the configuration, the experimental and theoretical moments of inertia and percentage difference.

3. The analysis of this interpretation should show that the moment of inertia of objects sharing an axis of rotation are additive.

For the ring configurations of Figures 3c and 3d, the total moment of inertia for the support+bar + ring configuration is $I_{tot} = I_{s+b} + I_{ring}$. For the single ring of Figure 3c use your previously calculated result in interpretation 2, and use $I_{ring} = I_{tot} - I_{s+b}$ and your results of interpretation 1 to determine the moment of inertia of the double ring configuration of Fig. 3d. The moment of inertia of objects sharing an axis of rotation are additive, so the moment of inertia of the double ring should be two times the moment of inertia of the single ring. According to your results, are the moments of inertia of objects sharing an axis of rotation additive? To substantiate your answer, determine the percentage difference between the moment of inertia of the double ring and twice the moment of inertia of the single ring.

4. The analysis of this interpretation should show that the moment of inertia increases as the distribution of mass moves farther away from the axis of rotation. You will compare the moments of inertia of the rings of configurations of Figures 3c and 3e.

   We have shown in interpretation 3 that the moments of inertia of objects sharing an axis of rotation are additive. This implies that the moment of inertia of one of the rings of Fig. 3e is half of that of the two rings, thus, use $I_{ring} = \frac{1}{2}(I_{tot} - I_{s+b})$ to determine the moment of inertia of one of the rings of Fig. 3e. The mass distribution of one ring of Fig. 3e is farther from the axis of rotation than that of the ring of Fig. 3c, is the moment of inertia of one ring of Fig. 3e greater than the moment of inertia of Fig. 3c?

   According to your previous comparison, how does the moment of inertia change as the distribution of mass moves farther away from the axis of rotation?