Newton's Second Law: An Introduction

1 Purpose

To verify Newton's 2^{nd} Law by showing that the acceleration of a body is proportional to the net external force on that body and that the acceleration is inversely proportional to the accelerated mass. To determine the force of friction between an object and surface and to show that the coefficient of static friction in greater than that of kinetic friction.

2 Theory

2.1 Newton's Second Law

Newton's Second Law states that the vector sum of the external forces on a body is equal to the mass times the acceleration of the body.

$$\sum_i \vec{F}_i = m\vec{a} \tag{1}$$

where $\vec{F}_i$ are the external forces on the body, m is the mass and $\vec{a}$ is the acceleration of the body.

Applying Newton's 2^{nd} law to the following situation,

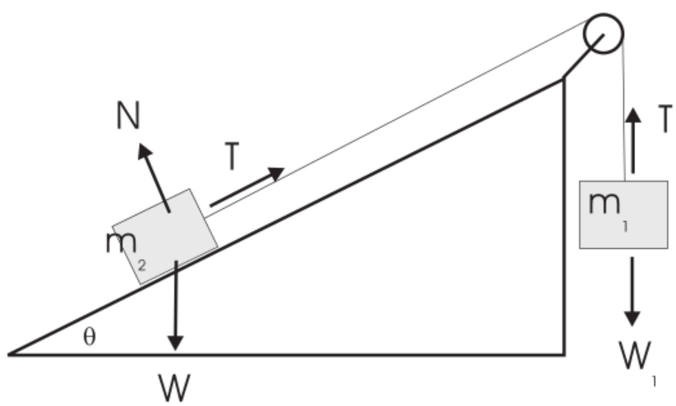

Figure 1: Two blocks and incline plane configuration

gives

$$a = \frac{m_1 - m_2 \sin\theta}{m_1 + m_2} g \tag{2}$$

for an horizontal surface, $\theta = 0$, which gives

$$a = \frac{m_1 g}{m_1 + m_2} \tag{3}$$

2.2 Friction

Attractive interactions between surfaces generate a force of friction opposite the direction of motion or against the applied force. The force of friction between two stationary objects in contact is the static friction and has a maximum of

$$f_{s,max} = \mu_s N \tag{4}$$

where $f_{s,max}$ is the maximum force of static friction, μ_s is the coefficient of static friction, and N is the normal force.

If the two objects in contact are moving relative to each other, the friction force is the kinetic friction and is given as

$$f_k = \mu_k N \tag{5}$$

where f_k is the force of kinetic friction, μ_k is the coefficient of kinetic friction. Typically the coefficient of static friction is greater than the coefficient of kinetic friction for two surfaces in contact.

3 Procedure

3.1 Newton's 2^{nd} law and force of kinetic friction

1. Assemble equipment as instructed by lab instructor.
2. Measure the mass of the cart on the horizontal surface (m_2).
3. For an incline plane with an angle of inclination equal to zero $\theta = 0°$ (frictionless horizontal surface), obtain the acceleration of the system of the hanging mass $m_1 =$ hanger $\approx 50g$ and cart mass $m_2 + 80g$. Measure the mass of the hanger and cart. The acceleration without friction is obtained by the slope of a velocity vs. time graph in Datastudio or Capstone.

4. Transfer 20 g (2 x 10g weights) from the cart m_2 to the hanging mass m_1 in order to keep $m_1 + m_2 =$ constant. Repeat the previous procedure for a total of 5 different values of m_1 and m_2. Fill-in Table 1.

Table 1			
m_1 (g)	m_2 (g)	$m_1 + m_2$ (g)	a (m/s^2)

5. For the horizontal frictionless surface, obtain the acceleration for 5 different m_2 but keeping $m_1 =$ hanger constant, and fill-in Table 2. Start with an empty cart and increase successively by 20 g.

Table 2			
m_1 (g)	m_2 (g)	$m_1 + m_2$ (g)	a (m/s^2)

3.2 Force of kinetic friction

6. Assemble equipment as instructed by lab instructor.

7. Obtain a velocity versus time curve for a block of wood of mass m_2 resting on a wood horizontal surface (the lab table) and hanging mass m_1 with friction between the block and table, where at one point of the movement, the mass m_1 reaches the ground. The mass m_1 should be large enough to overcome the static friction. Note the position of m_2 where m_1 reaches the ground. Use the velocity vs time graph to determine the maximum velocity of m_2. This corresponds to the point where m_1 reaches the ground and has no effect on m_2. Note the final position of m_2 when stopped. Fill-in the first row of Table 3.

8. Increase the mass m_2 by 50 g and repeat the previous procedure for a total of 5 different values of m_2. You may need to tape the weights added to the block of wood. Fill-in Table 3.

Table 3	$x_i(cm)=$	
m_2 (g)	v_{max} (m/s)	x_f (cm)

3.3 Force of static friction

9. With the wooden block (m_2) at rest on the horizontal wood surface of the lab table, replace the hanging mass m_1 by a spring balance. Make sure the string is parallel to the lab table surface. With the weight of the spring balance supported by your hand, adjust the spring balance to indicate 0, and let the balance hang. Take a video, or watch the spring balance while gently pulling down on the balance until the static friction is exceeded and the cart (m_2) starts moving. Use your video, or by eye, to determine the maximum force (in grams) exerted. Repeat 3 times. Fill-in the first row of Table 4.

10. Increase the mass of m_2 by 50 g and repeat the previous procedure for a total of 5 different values of mass m_2. Fill-in Table 4.

Table 4			
m_2 (g)	$f_{s,max\ 1}$ (g)	$f_{s,max\ 2}$ (g)	$f_{s,max\ 3}$ (g)

4 Interpretation of Results

1. Plot a graph of a versus F_{net} using the values of Table 1 and $F_{net} = m_1 g$. Use a value of g to be 9.785 N/kg. Convert your gram units to kilogram. All variables are scalar quantities. Use linear regression and find the best-fit slope. Compare (using the percentage difference formula) the best-fit slope to $1/(m_1 + m_2)$. A straight line shows that $a \propto F_{net}$ and a slope equal to $1/(m_1 + m_2)$ validates Newton's 2^{nd} law. How do you interpret your result?

2. Plot a graph of a versus $1/(m_1 + m_2)$ using Table 2. Convert your gram units to kilogram. All variables are scalar quantities. Use linear regression and find the best-fit slope. Compare (using the percentage difference formula) the best-fit slope to F_{net}. A straight line shows that $a \propto 1/(m_1 + m_2)$ and a slope equal to F_{net} validates Newton's 2^{nd} law. How do you interpret your result?

3. The deceleration due to friction is calculated using the third kinematic equation and the maximum speed given on the velocity versus time graph in Datastudio or Capstone, and the distance to stop ($\Delta x = x_f - x_i$) using values of Table 3: $a_{kf} = \frac{(v_{max})^2}{2\Delta x}$. Calculate a_{kf}. Determine the force of kinetic friction and normal force generated between the two wooden surfaces using the values of Table 3, where $f_k = m_2\ a_{kf}$ and $F_N = m_2\ g$, plot a graph of f_k vs. F_N. Use a value of g to be 9.785 N/kg. All variables are scalar quantities. Use linear regression to obtain the slope which is equal to the coefficient of kinetic friction μ_k of the two wooden surfaces in contact.

4. Average your values for the force of static friction (across each row) for each value of m_2 of Table 4 and convert the force from grams to newtons using $f_{s,max}[N] = f_{s,max}[g] \times \frac{9.785[N]}{1000[g]}$. Plot a graph of $f_{s,max}$ vs. F_N, where $F_N = m_2\ g$ is the normal force. All variables are scalar quantities. Use linear regression to obtain the slope which is equal to the coefficient of static friction μ_s of the two wooden surfaces in contact.

5. Do your results confirm that the coefficient of kinetic friction is less than the coefficient of static friction?

6. Google kinetic and static coefficients of friction for wood on wood, you may average values if you find different values for different references. Compare (% difference) your experimental coefficients of static and kinetic friction values to those cited in the internet. Give your references.