Position, Velocity, and Acceleration

1 Purpose

To use the kinematic equations to determine where two objects must initially be positioned such that a collision can occur between two objects. One object is a ball falling vertically and the other object is a cart moving in a straight line down a frictionless incline plane. This result will be verified experimentally.

2 Theory

The following kinematic equations may be used for the experiment.

$$\vec{x}_f = \vec{x}_i + \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2 \tag{1}$$

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t \tag{2}$$

$$v_f^2 = v_i^2 + 2\vec{a}\cdot\Delta\vec{x} \tag{3}$$

An object **sliding** down an incline plane will have a theoretical acceleration magnitude of

$$a = |\vec{a_g}|\sin\theta \tag{4}$$

where $|\vec{a_g}|$ is the magnitude of the acceleration due to gravity and θ is the angle of inclination of the incline plane. For an object in free fall, the magnitude of the acceleration of the object is simply

$$a = |\vec{a_g}| \tag{5}$$

3 Procedure

1. Set-up the equipment as indicated by your instructor. The experiment will use three photogates, the first photogate is used to trigger the ball to fall, and the second and third photogate are used to determine the acceleration of the cart down the incline plane.

2. Turn-off the air supply to the air track and move the cart beneath the drop-box device such that the cart basket can capture the falling ball. Read the position (x_f) of the downward (left) edge of the cart. Move the cart from right to left triggering the photogate just to the right of where the ball falls. Record the initial position (x_i) of the downward (left) edge of the cart when it triggers the photogate. Fill-in Table 1 of the data sheet.

3. Adjust the angle of inclination such that when the cart is fully up the incline plane and the drop box is in the lowest position, the ball drops into the cart basket when triggered by the first photogate. Read the downward (left) edge of the cart which corresponds to the initial position (x) of the cart. Repeat for a total of three (3) times and record the initial position of the cart for each time. Measure the vertical distance (Δy) between the bottom of the hanging ball to the top surface of the cart. Fill-in Table 1 of the data sheet.

4. Repeat for two additional (mid and high) drop box positions. Fill-in Table 1 of the data sheet.

5. Support a square metal bar horizontally with one end against the top edge of the air track and measure using a vernier caliper the vertical height of the other end over the air track. Measure the length of the square bar. Fill-in Table 2 of the data sheet.

6. Use an inclinometer app on your smartphone to measure the inclination of the incline plane. You need to calibrate the inclinometer using a leveled horizontal surface. Fill-in Table 2 of the data sheet.

7. In this procedure, the data to calculate the acceleration of the cart sliding down the incline plane will be obtained. Here you will measure the time it takes the cart to slide down the incline plane from the top most position to 7 different positions down the incline plane. You will need to move the second photogate to seven different positions. First, read the position of the downward (left) edge of the cart (x_i) when it is at the top-most position and then read the position (x_f) downward (left) edge of the cart where it activates the LED of the second photogate. Use the program capstone to measure the time it takes the cart to slide from rest to the second photogate. Record the time. Then move the photogate down the incline plane around 5-10 cm, the distance that you move is not critical, but measure accurately the position of the second photogate. Fill-in Table 3 of the data sheet.

Data and observations

Group number: _______

Group members: ____________________________
____
____
____

Table 1	$x_i =$		$x_f =$	
//////	Δy (cm)	x_1 (cm)	x_2 (cm)	x_3 (cm)
//////				
//////				
//////				

Table 2	bar length =		bar height =		θ_{sphone} =	

Table 3	x_i	x_{1f}	x_{2f}	x_{3f}	x_{4f}	x_{5f}	x_{6f}	x_{7f}
x (cm)								
t (s)								

4 Interpretation of Results

1. Plot a graph of the magnitud of the position ($|\vec{x}|$) versus time squared (t^2). Do a linear regression and include the linear equation and the R^2 value on the graph. Basically we are linearizing a quadratic equation. The position of the cart with initial velocity of zero is given by

$$\vec{x}_f = \vec{x}_i + \frac{1}{2}\vec{a}(\Delta t)^2$$

   by plotting x_f vs t^2 we are converting the above equation to y = m x + b, where the slope m is equal to $\frac{1}{2}a$ and the y-intercept b is x_i. Determine the acceleration of the cart on the frictionless incline plane.

2. Determine using trigonometry the inclination of the air track. Average the trigonometric value and the smartphone value and calculate the theoretical acceleration magnitude of the cart based on inclination using $a = a_g \sin\theta$ where $a_g = 9.785$ m/s^2 and θ is the angle of inclination. Compare this acceleration to the experimental acceleration determined in interpretation 1 using the percentage difference.

$$\%\text{ difference} = \frac{|A - B|}{\frac{A+B}{2}} \times 100 \tag{6}$$

3. Determine the time it takes the metal ball to fall your measured Δy. Use $\Delta y = 0.5\, a_g\, (\Delta t)^2$ and solve for Δt. Remember $a_g = 9.785\, m/s^2$.

4. Using $\Delta x = v_i\, \Delta t + 0.5\, a\, (\Delta t)^2$ solve for v_i, v_i is the velocity of the cart when it triggers the photogate. Be careful with the signs of the physical quantities as they represent vectors and can be positive or negative. Since the cart is moving towards the left, v_i should be negative.

5. Using $v_f^2 = v_i^2 + 2\, a\, \Delta x$ calculate the Δx for your cart starting at rest ($v_i = 0$) at its initial position to the photogate triggering the ball. v_f is the velocity calculated in the previous imterpretation.

6. Calculate the initial position of your cart, according to the above kinematic equations, using the photogate position and the Δx calculated in the previous interpretation. Remember, $\Delta x = x_f - x_i$. Compare this calculated position to your experimental value using the percentage difference .

7. Repeat interpretations 3 - 6 for the mid and high positions of the dropbox.