The Biot-Savart Law

1 Purpose

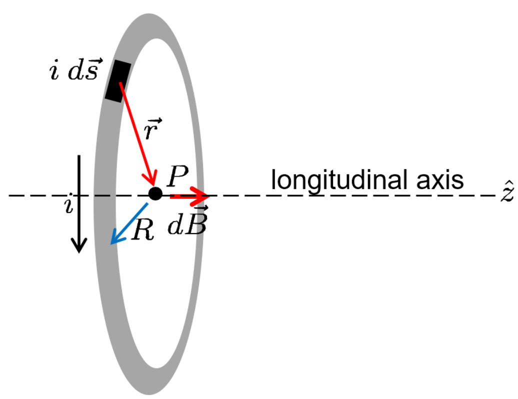
To determine the magnetic field along the longitudinal axis of two Helmholtz coils. This experimental result will be compared to a theoretical result obtained with the Biot-Savart Law. The horizontal component of the Earth's magnetic field will also be determined.

2 Theory

The Biot-Savart Law states that the differential magnetic field $d\vec{B}$ at a point P in space due to a current element $id\vec{s}$ is given by

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\vec{r}}{r^3} \tag{1}$$

Applying Biot-Savart Law to the longitudinal axis of a coil consisting of N-turns of wire as shown in the following figure

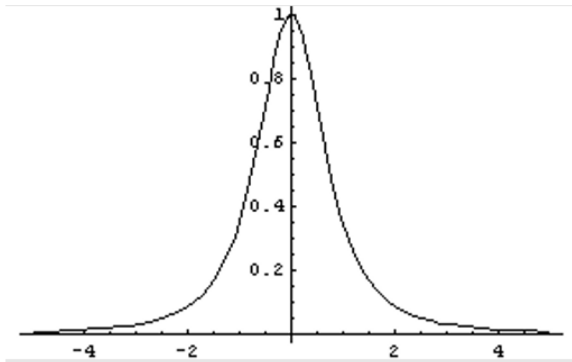


gives that

$$B_{coil} = \frac{N\mu_o iR^2}{2(z^2+R^2)^{3/2}} \tag{2}$$

where μ_o is the permeability of free space $\mu_o = 4\pi\times10^{-7}$Tm/A, i is the current in the coil, R is the radius of the coil, and z is the distance along the longitudinal axis.

A plot of B_{coil} versus z with all variables normalized to 1 gives



The Helmholtz coil configuration consists of two coils separated by the coil radius R with the current in each coil going in the same direction. The total magnetic field along an axis of both coils is then given by the sum of the magnitudes of the B-field of each coil.

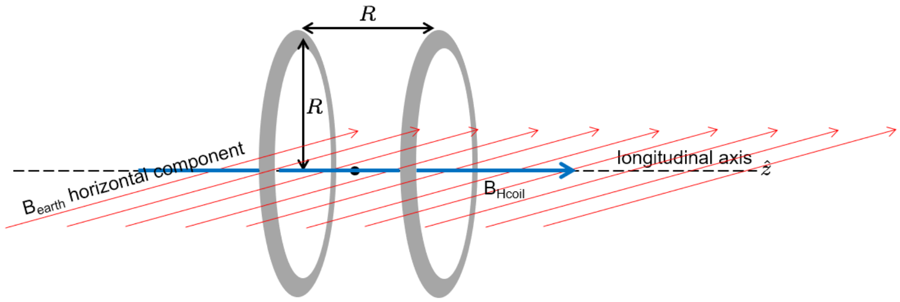
The total magnetic field for two coils separated by distance R, where one coil is at z=-R/2 and the other is at z=+R/2 is given by

$$B_{coil,calc} = \frac{N\mu_o i R^2}{2([z+\frac{R}{2}]^2 + R^2)^{3/2}} + \frac{N\mu_o i R^2}{2([z-\frac{R}{2}]^2 + R^2)^{3/2}} \tag{3}$$

The maximum magnetic field in the center of two Helmholtz coils is given by

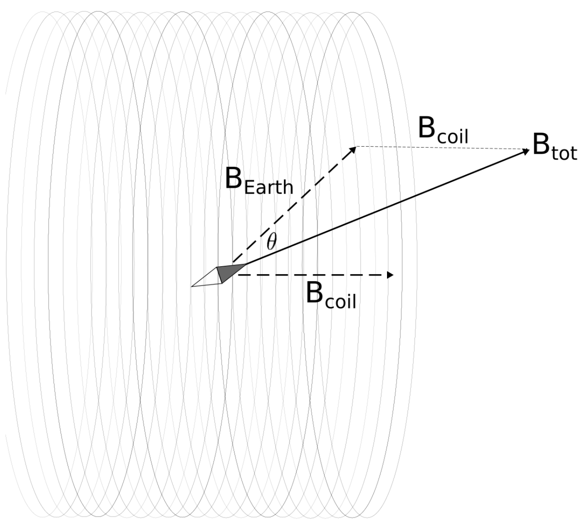
$$B_{coil,max} = \frac{8}{\sqrt{125}}\frac{N\mu_o i}{R} \tag{4}$$

In this experiment, the magnetic deflection of a compass due to the combined effect of a perpendicular external magnetic field and that of the Earth's will be used to measure the external magnetic field.



The total magnetic field is given as the vector sum of the horizontal component of Earth's magnetic field and the coil's magnetic field, where

$$\vec{B}_{tot} = \vec{B}_{coil} + \vec{B}_{Earth,hor} \tag{5}$$



and where,

$$\tan\theta = \frac{B_{coil,exp}}{B_{Earth,hor}} \tag{6}$$

where the θ is the compass deflection angle between the magnetic north direction and B_{tot}.

For a deflection angle of 45 degrees,

$$B_{coil,exp} = B_{coil,max} = B_{Earth,hor} \tag{7}$$

such that

$$\frac{B_{coil,exp}}{B_{Earth,hor}} = \frac{B_{coil,exp}}{B_{coil,max}} = \tan\theta \tag{8}$$

3 Procedure

1. Connect the equipment as indicated by your lab instructor.

2. Record the radius R and number of turns N of the Helmholtz coils.

3. Align the longitudinal axis of the coils perpendicular to the magnetic north-south direction, one of the lateral wires of the magnetic needle should point to zero.

4. Add a little positive current to establish the direction of a positive deflection.

5. With NO current, measure the compass deflection angle from $z = -40$ cm to -5 cm at each 5 cm, from -5 cm to 5 cm at each 1 cm, and from 5 cm to 40 cm at each 5 cm. Include a negative sign if the deflection is in the negative direction.

6. Adjust the current such that the deflection angle on the compass in the center of the two coils is +45 degrees.

7. Measure the compass deflection angle from $z = -40$ cm to -5 cm at each 5 cm, from -5 cm to 5 cm at each 1 cm, and from 5 cm to 40 cm at each 5 cm. Continually adjust the voltage to maintain the current of procedure 6. Include a negative sign if the deflection is in the negative direction.

8. Invert the current direction in the coils and **repeat the previous two procedures**. The deflection angles will now be negative and include the negative sign if in the negative direction.

9. Organize your data in a table as the following

ruler	center	R (cm) =				N =		
		NO current		+ curr. (mA) =		- curr. (mA) =	
		z-pos. (cm)	defl. (^{o})	z-pos. (cm)	+ defl. (^{o})	z-pos. (cm)	- defl. (^{o})
4.4	10	-40		-40		-40	
9.4	15	-35		-35		-35	
14.4	20	-30		-30		-30	
19.4	25	-25		-25		-25	
24.4	30	-20		-20		-20	
29.4	35	-15		-15		-15	
34.4	40	-10		-10		-10	
39.4	45	-5		-5		-5	
40.4	46	-4		-4		-4	
41.4	47	-3		-3		-3	
42.4	48	-2		-2		-2	
43.4	49	-1		-1		-1	
44.4	50	0		0		0	
45.4	51	1		1		1	
46.4	52	2		2		2	
47.4	53	3		3		3	
48.4	54	4		4		4	
49.4	55	5		5		5	
54.4	60	10		10		10	
59.4	65	15		15		15	
64.4	70	20		20		20	
69.4	75	25		25		25	
74.4	80	30		30		30	
79.4	85	35		35		35	
84.4	90	40		40		40	

4 Interpretation of Results

1. Prepare a table of values consisting of 5 columns: position z, average deflection $\theta_{ave} = \frac{\theta_+ - \theta_-}{2}$, $\tan\theta_{ave}$, $\frac{B_{coil,calc}}{B_{coil,max}}$, and $B_{coil,max}\cos\theta_{nocurr.}$. Use the average of the current magnitudes in amperes [A] in your calculations. $\frac{B_{coil,calc}}{B_{coil,max}}$ is calculated using equations 3 and 4. Convert your distances R and z to meters [m]. Give examples of your calculations.

2. Plot $B_{coil,max}\cos\theta_{nocurr.}$ vs position z and manually adjust the y-scale of the graph such that the minimum vaue is zero. Comment on the uniformity of the horizontal component of the Earth's magnetic field over 0.8 meter of distance on your work bench. The variance (google it!) of the horizontal component of the Earth's magnetic field may help to justify your comment.

3. Plot the experimental curve of $\frac{B_{coil,exp}}{B_{coil,max}}$ which is equal to $\tan\theta_{ave}$ using X-Y scatter points and the theoretical curve $\frac{B_{coil,calc}}{B_{coil,max}}$ using smooth curved lines vs position z. Superimpose both curves on the same graph.

4. Calculate $\sqrt{(\frac{B_{coil,calc}}{B_{coil,max}} - \frac{B_{coil,exp}}{B_{coil,max}})^2}$ at each position and average the results. A value of zero gives a perfect correlation between the experiment and calculated values while a value of 1 gives no correlation. Give your opinion on how good (or bad) is the correlation between the experimental and the theoretical curve?

5. Calculate the Earth's horizontal component of the magnetic field for positions -5 cm to 5 cm. Use $B_{Earth,hor} = \frac{B_{coil,calc}}{\tan\theta}$. Average your results to give your EXPERIMENTAL value.

6. Determine the ACCEPTED Earth's horizontal component of the magnetic field of your laboratory (use Google Earth to find the lat-long and elevation of the 200 building) using the on-line World Magnetic Model (WMM) at

   `http://www.ngdc.noaa.gov/geomag-web/#igrfwmm`

7. Compare your EXPERIMENTAL value to the ACCEPTED value using the percent difference formula.