The Capacitance of a Parallel Plate Capacitor

1 Purpose

Circular and rectangular parallel plate capacitors will be used. We will verify that the capacitance of the parallel plate capacitor is proportional to the area of the plate and inversely proportional to the plate separation. We will also determine the dielectric constant of air, PVC foamboard, cork covered fiberboard, and parchment paper.

2 Theory

A capacitor consists of two conductors relatively close to each other and separated by an insulating material of permittivity ϵ. Since it requires work to transfer charges from one conductor to the other, the total charge on each conductor will be proportional to the potential difference (voltage) between the conductors,

$$q \propto \Delta V.$$

For the parallel plate it is relatively easy to derive the exact relationship between stored charge and voltage across the plates. The proportionality constant is defined as the capacitance and is given the symbol C. The capacitance depends on the geometry (physical arrangement) and the dielectric material.

For the parallel plate capacitor, Gauss' Law yields that the charge q on a plate is

$$q = \kappa \epsilon_0 E A,$$

where κ is the dielectric constant of the insulating material and ϵ_0 is the permittivity of free space, and A is the area of the plate. The potential difference between the plates is given by

$$\Delta V = \int_{+}^{-} E ds = \int_{0}^{d} \frac{q}{\kappa \epsilon_0 A} ds$$

$$\Delta V = \frac{qd}{\kappa \epsilon_0 A}$$

or

$$q = (\frac{\kappa \epsilon_0 A}{d}) \Delta V = (\frac{\epsilon A}{d}) \Delta V$$

The capacitance for the parallel plate capacitor is $C = \frac{\epsilon A}{d} = \frac{\kappa \epsilon_0 A}{d}$.

In practice, it is difficult to isolate electrically the capacitor from other conductors, such that, in addition to the parallel plate capacitance, we may have some stray capacitance $C = C_{pp} + C_{stray}$.

The dielectric constant κ is defined as $\kappa = \frac{C}{C_0} = \frac{\epsilon}{\epsilon_0}$, where the C is the capacitance with the dielectric material and C_0 is the capacitance in vacuum (or air). The effective dielectric for a multilayered dielectric is determined by considering series capacitors and is given by

$$\frac{d}{\kappa_{eff}} = \sum_i \frac{d_i}{\kappa_i} \tag{1}$$

where $d = \sum_i d_i$.

The equivalent capacitance of capacitors in parallel is given by:

$$C_{eq} = C_1 + C_2 + C_3 + ... + C_n \tag{2}$$

and the equivalent capacitance of capacitors in series is given by:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n} \tag{3}$$

3 Procedure

3.1 Air dielectric

1. In this section we will be using the Pasco circular parallel plates with area $A = \pi r^2$. Trace the contour of one of the plates on a piece of paper and measure the diameter of the circle.

2. Use the LCR meter in parallel capacitance mode to measure the capacitance of the parallel plate capacitor for various separations, from d = 0.1 cm, increasing every 0.1

cm up to 1 cm, and d = 2, 3, 4, and 5 cm. Move, if possible, obvious conductors nearby, and don't touch the plates or wires.

3.2 PVC foamboard dielectric

3. In this section we will be using rectangular parallel plates with area $A = l \times w$. Measure the length and width of the plates.

4. Place one of the conducting plates flat on the wooden lab bench, conductor facing up. Add one PVC foamboard on top, and cover it with the other conducting plate, conductor facing down. Add a smaller thick piece of PVC on top with a weight to press the sandwiched PVC foamboard. Use the LCR meter to measure the parallel capacitance of the parallel plate capacitor with the PVC foamboard for various separations, from d = 1 board, 2 boards, etc. up to 5 PVC foamboards. Also record the parallel resistance given by the meter, if OL is listed, that means infinite resistance. Use a vernier caliper to measure the thickness of 5 PVC foamboards.

5. Separate two (2) PVC foamboards by placing PVC spacers in the corners, and inserted just enough to support the top PVC board. These two boards with an air gap will be placed between the two parallel plate conductors. Add on top only the thick PVC piece. Use the LCR meter to measure the parallel capacitance of this particular configuration.

6. Separate the two parallel plate conductors using three layers of PVC spacers inserted just enough in the corners, to support the top conducting plate. Use the LCR meter to measure the parallel capacitance of this particular configuration.

7. Measure the parallel capacitance of the two parallel conducting plates sandwiching three layers of parchment paper. Add a smaller thick piece of PVC on top with a weight to press the sandwiched paper. Use a ruler to measure the dimension of one of the conductors. This data belongs to the Parchment paper dielectric subsection.

3.3 Cork-covered fiberboard dielectric

8. In this section we will be using rectangular parallel plates with area $A = l \times w$. Measure the length and width of the foil that is taped to the cork-covered fiber boards.

9. Place one of the conducting plates flat on the wooden lab bench, conductor facing up. Add one cork-fiber board on top, and cover it with the other conducting plate, conductor facing down. Add a thick piece of PVC on top with a weight to press the sandwiched cork-fiber board. Use the LCR meter to measure the parallel capacitance of the parallel plate capacitor with the cork-fiber board for various separations, from d = 1 board, 2 boards, etc. up to 5 cork-fiber boards. Also record the parallel resistance given by the meter, if OL is listed, that means infinite resistance.

10. Use a vernier caliper to measure the thickness of 5 cork-fiber boards. Also measure the thickness of the two cork layers and the fiber layer, these should add up to the average thickness of one board.

3.4 Parchment paper dielectric

11. Use the LCR meter to measure the parallel capacitance of four different parallel plate capacitors of different areas with three (3) layers of parchment paper as the dielectric material. Add a smaller thick piece of PVC on top with a weight to press the sandwiched paper. Also record the parallel resistance given by the meter, if OL is listed, that means infinite resistance. Use a ruler to measure the dimensions of one of the conductors of each capacitors. Remember that you have the capacitance and dimensions of the larger conducting plate in the PVC section.

12. Fold a piece of parchment paper at least 4-5 times and measure the thickness using a micrometer. Record the quantity of folds and thickness.

3.5 Series and parallel capacitors

13. Use a multimeter of LCR meter to measure the capacitance of the three parallel plate capacitors separately.

14. Connect the three parallel plate capacitors in parallel (all of the top wires together and all of the bottom wires together) to a breadboard and then to the multimeter or LCR meter. Measure the parallel capacitance.

15. Connect the three parallel plate capacitors in series (top wire of one capacitor and the bottom wire of the next capacitor together on the breadboard and successively with

all the capacitors). Connect the bottom wire of the first capacitor and the top wire of the third capacitor to the multimeter or LCR meter. Measure the parallel capacitance.

4 Interpretation of Results

1. (a) Using your parallel plate capacitor with air dielectric, plot the capacitance C vs. (1/d). C should be in [F] and d in [m]. A straight line shows that the capacitance is inversely proportional to the separation d. Use linear regression and the area of the plates to determine the dielectric constant of air. Make sure you format the trendline label to increase the number of significant figures. Compare, using the percentage error formula $\%\text{error} = \frac{|A-B|}{A} * 100$, your experimental air dielectric constant to the accepted value of 1.00059.

   (b) The stray capacitance which is the capacitance due to nearby conductors, wires, meters, and human bodies, etc. is given by the y-intercept. How much is the stray capacitance in pF?

2. (a) Using your parallel plate capacitor with PVC foamboard dielectric, plot the capacitance C vs. (1/d). C should be in [F] and d in [m]. A straight line shows that the capacitance is inversely proportional to the separation d. Use linear regression and plate area to determine the dielectric constant of the PVC foamboard. Make sure you format the trendline label to increase the number of significant figures. Compare, using the percentage difference formula, your experimental PVC foamboard dielectric constant to a value found on the internet, as a starting point, google 'dielectric constant PVC foam'.

   (b) How much is the stray capacitance in pF?

   (c) Determine the experimental value of the effective dielectric constant of the PVCboard-air-PVCboard configuration using your measured capacitance minus the stray capacitance with and without the PVCboard-air-PVCboard layers, such as,

$$\kappa_{\text{eff,exp}} = \frac{C_{\text{PVC}-\text{air}-\text{PVC}} - C_{\text{stray}}}{C_{\text{air}-\text{air}-\text{air}} - C_{\text{stray}}}$$

(d) Calculate the theoretical effective dielectric constant using the effective dielectric equation (1) from the theory, adapted to the PCV-air-PVC configuration

$$\frac{3}{\kappa_{\text{eff}}} = \frac{1}{\kappa_{\text{PVC}}} + \frac{1}{\kappa_{\text{air}}} + \frac{1}{\kappa_{\text{PVC}}}$$

using the accepted dielectric constant for air and of the experimental dielectric constant of PVCboard of interpretation 2a. How does it compare (quantify your comparison using the % difference) to your experimental value determined in interpretation 2c?

3. (a) Using your parallel plate capacitor with cork-fiberboard dielectric, plot the capacitance C vs. (1/d). C should be in [F] and d in [m]. Use linear regression and the plate area to determine the effective dielectric constant of the cork-fiberboard. Make sure you format the trendline label to increase the number of significant figures.

   (b) How much is the stray capacitance in pF?

   (c) Calculate the theoretical effective dielectric constant (using the effective dielectric equation from the theory) of the multilayered cork and fiber board using your measured thicknesses of the cork and fiber and using the dielectric constant of cork ($\kappa = 1.7$) and fiber ($\kappa = 5$). How does it compare (quantify your comparison using the % difference) to your experimental value of interpretations 3a? Use the following ideas to explain the difference, if necessary. If your boards register a parallel resistance that is not infinite, this means that the material has absorbed some moisture. The dielectric constant of water is approximately 80.

4. (a) Using your parallel plate capacitors with paper dielectric, plot the capacitance C vs. A using your five different conducting plate sizes. C should be in [F] and A in [m^2]. A straight line shows that the capacitance is proportional to area A. Calculate the thickness of one sheet of parchment paper using $d = \frac{\text{total thickness}}{2^n}$ where n is the number of folds. Use linear regression and thickness of three (3) sheets of parchment paper to determine the dielectric constant of the parchment paper.

(b) How much is the stray capacitance in pF?

(c) Has the parchment paper absorbed some moisture? How does it compare to the cork-fiber board? And what about the PVC foam boards?

(d) Compare your experimental paprchment paper dielectric constant to the accepted value for paper found on the internet, as a starting point, google 'dielectric constant clippercontrols', look at different sources, you can even ask ChatGPT, provide your references.

5. Calculate the series and parallel equivalent capacitance of your three parallel plate capacitors using equations 2 and 3 and compare using % difference to the experimental values.