# Moment of Inertia

## 1 Purpose

To determine experimentally and theoretically the moments of inertia of a disk and rings of definite thickness about various rotational axes. To show that the moment of inertia depends on the geometry of the rotating object, the moment of inertia of objects sharing an axis of rotation are additive, that the moment of inertia increases as the distribution of mass moves farther away from the axis of rotation, to verify the parallel axes theorem, and to study the relationships between torque and rotational motion.

## 2 Theory

Newton's second law for rotation is given as

$$\sum \tau = I\alpha \tag{1}$$

where $\tau$ are the torques on the rotating object, $I$ is the moment of inertia of the rotating object, and $\alpha$ is the angular acceleration of the rotating object.

The moment of inertia $I$ plays essentially the same role in rotation as the mass $m$ does in translational motion. The moment of inertia is the property of a body to resist changes of its rotational state about a rotational axis.

### 2.1 Review of rotational motion

| Rotation Equations | Relation to Linear Motion |
|---|---|
| $\omega = d\theta/dt$ | $v = \omega r$ |
| $\alpha = d\omega/dt = d^2\theta/dt^2$ | $a = \alpha r$ |
| $\tau = r_\perp F$ | |
| $\tau = I\alpha$ | $F = ma$ |

From the diagram below we see that the applied torque $\tau_a$ on the rotating platform is due to the tension $T$ of the string that is wrapped around the axis of rotation.

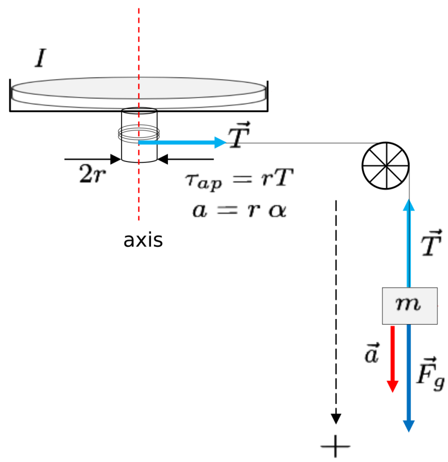


Figure 1

The net torque $\tau_{net}$ on the rotating platform is given by

$$\tau_{net} = \tau_a - \tau_f \tag{2}$$

where $\tau_f$ is a frictional torque. The net torque of the rotating platform is also equal to

$$\tau_{net} = I\alpha. \tag{3}$$

Manipulating the above two equations gives

$$\tau_a = I\alpha + \tau_f \tag{4}$$

The applied torque $\tau_a$ can be calculated as

$$\tau_a = rT \tag{5}$$

and the angular acceleration $\alpha$ can be obtained from the linear acceleration as

$$\alpha = a/r \tag{6}$$

Using Newton's second law, the net force on the hanging weight is

$$F_{net} = mg - T = ma$$

$$T = mg - ma \tag{7}$$

such that the tension $T$ can also be calculated using the acceleration.

The moment of inertia $I$ is actually the sum of all of the moments of inertia of each component bodies, i.e.

$$I = I_s + I_a \tag{8}$$

where $I_s$ is the moment of inertia of the rotating support and $I_a$ is the moment of inertia of the accesory body.

## 2.2 Parallel Axis Theorem

The parallel axis theorem states that the moment of inertia of any body about an axis (not passing through the CM) is

$$I = I_{cm} + Md^2 \tag{9}$$

where $I_{cm}$ is the moment of inertia of the body about an axis passing through its center of mass, $M$ is the mass of the body, and $d$ is the distance between the the two parallel axes.

The moment of inertia of a disk and rings with respect to different axes orientation are given in the following figure.

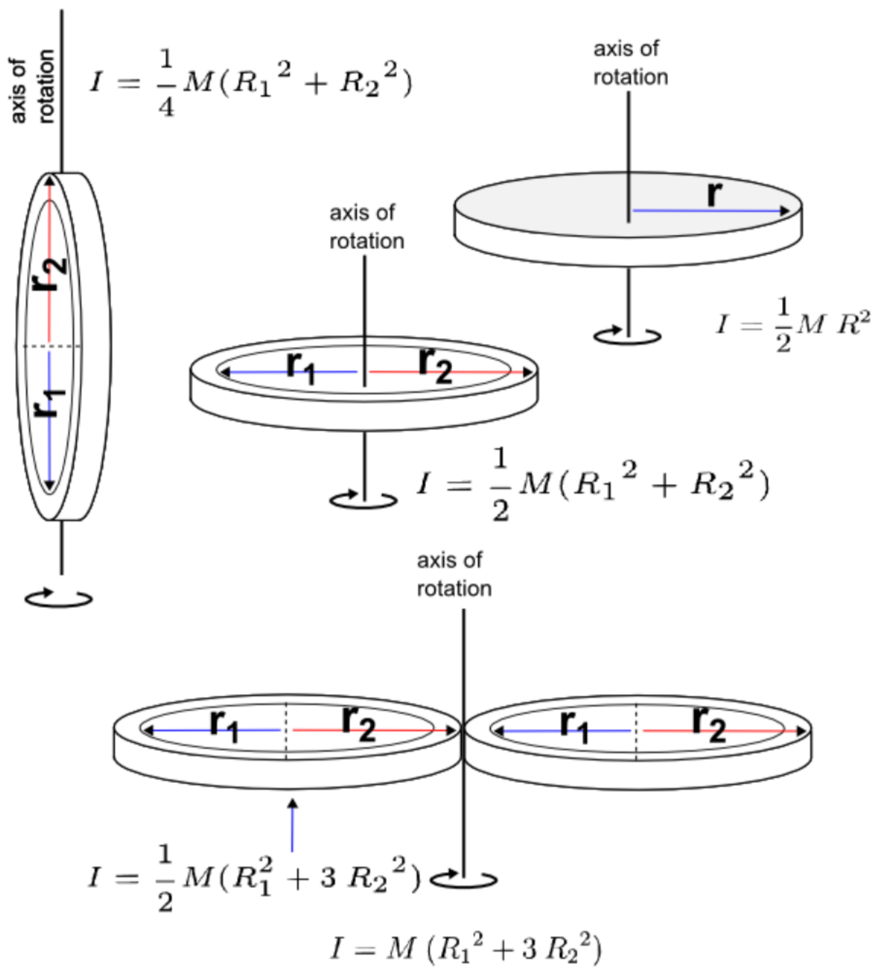


Figure 2

# 3 Procedure

1. Assemble equipment as directed by your lab instructor. The long bar will be mounted on the rotary support using two screws.

2. Measure:

    - the diameter of the rotary support axis
    - the mass and diameter of the disk
    - the diameter of the inner and outer edge of both cylindrical rings
    - the mass of both cylindrical rings

| | |
|---|---|
| hanger (g) | |
| diameter rotary support (cm) | |
| mass disk (g) | |
| diameter disk (cm) | |
| mass ring 1 (g) | |
| outer diameter ring 1 (cm) | |
| inner diameter ring 1 (cm) | |
| mass ring 2 (g) | |
| outer diameter ring 2 (cm) | |
| inner diameter ring 2 (cm) | |

3. Measure the acceleration of 5 different hanging weights, hanger ($\approx$ 50g) + up to 6 x 20g weights, using the smart pulley arrangement for the following configurations:

    - rotary support with attached long bar (3a) [hanger + 4 x 20g]

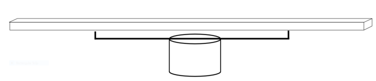

Figure 3a

    - rotary support with attached long bar with **single disk** (3b) mounted horizontally [(hanger + 20g) + 4 x 20g]

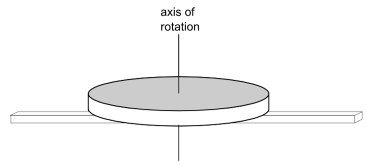


Figure 3b

- rotary support with attached long bar with **single ring** (3c) mounted horizontally [(hanger + 20g) + 4 x 20g]

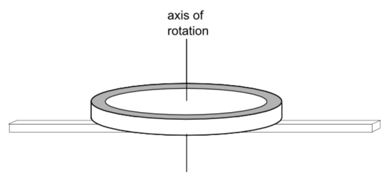


Figure 3c

- rotary support with attached long bar with **two rings** (3d) mounted horizontally one on top of the other [(hanger + 40g) + 4 x 20g]

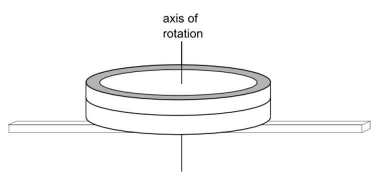


Figure 3d

- rotary support with attached long bar with **two rings** (3e) mounted horizontally side by side [(hanger + 40g) + 4 x 20g]

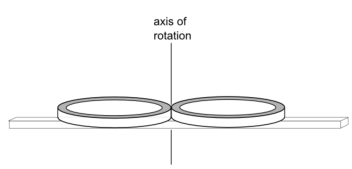


Figure 3e

- rotary support with attached long bar with a **single ring** (3f) mounted vertically [(hanger + 20g) + 4 x 20g]

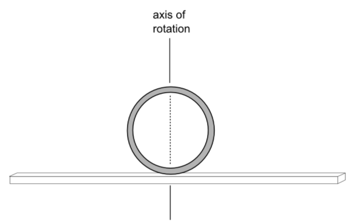


Figure 3f

4. Arrange your data as such:

| To measure | | | To calculate | | |
|---|---|---|---|---|---|
| Configuration | hanging mass m (g) | a ($m/s^2$) | T (N) | $\tau_a$ (Nm) | $\alpha$ ($rad/s^2$) |
| support+bar Fig. 3a | | | | | |
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| support+bar Fig. 3a | | | | | |
| support+bar Fig. 3a | | | | | |
| support+bar Fig. 3a | | | | | |
| support+bar + 1 horizontal disk Fig. 3b | | | | | |
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| support+bar + 1 horizontal disk Fig. 3b | | | | | |
| support+bar + 1 horizontal disk Fig. 3b | | | | | |
| support+bar + 1 horizontal ring Fig. 3c | | | | | |
| support+bar + 1 horizontal ring Fig. 3c | | | | | |
| support+bar + 1 horizontal ring Fig. 3c | | | | | |
| support+bar + 1 horizontal ring Fig. 3c | | | | | |
| support+bar + 1 horizontal ring Fig. 3c | | | | | |
| support+bar + 2 horizontal rings Fig. 3d | | | | | |
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| support+bar + 2 horizontal rings Fig. 3d | | | | | |
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| support+bar + 2 horizontal rings Fig. 3e | | | | | |
| support+bar + 2 horizontal rings Fig. 3e | | | | | |
| support+bar + 1 vertical ring Fig. 3f | | | | | |
| support+bar + 1 vertical ring Fig. 3f | | | | | |
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| support+bar + 1 vertical ring Fig. 3f | | | | | |

# 4 Interpretation of Results

1. For each configuration, plot $\tau_a$ vs. $\alpha$ and give a table of values for each graph. Use linear regression to find the best-fit slope. The slope of your linear regression gives the TOTAL moment of inertia. Summarize your total moments of inertia and configurations in a table.

2. The analysis of this interpretation should show that by changing the geometry (and mass distribution) of the rotating object, the moment of inertia will also change. Compare the moment of inertia of the ring to the moment of inertia of the disk. The mass of the disk is uniformly distributed from the center to the edge giving a smaller moment of inertia to that of a ring of similar mass and diameter. The total moment of inertia for the support+bar + disk configuration is $I_{tot} = I_{s+b} + I_{disk}$. Use $I_{disk} = I_{tot} - I_{s+b}$ and your results of interpretation 1 to determine the moment of inertia of the disk of Fig. 3b. Use $I_{ring} = I_{tot} - I_{s+b}$ and your results of interpretation 1 to determine the moment of inertia of the ring of Fig. 3c. Verify that the ratio of $\frac{I_{ring}}{I_{disk}} \frac{m_{disk}}{m_{ring}}$ is slightly less than 2. It should not give exactly two as the ring also has an inner diameter smaller than the outer diameter such that some of the mass is distributed towards the center. The mass ratio $\frac{m_{disk}}{m_{ring}}$ is used to compensate for the mass difference between disk and ring.

3. The analysis of this interpretation should show that the moment of inertia of objects sharing an axis of rotation are additive. For the ring configurations of Figures 3c and 3d, the total moment of inertia for the support+bar + ring configuration is $I_{tot} = I_{s+b} + I_{ring}$. Use $I_{ring} = I_{tot} - I_{s+b}$ and your results of interpretation 1 to determine the moment of inertia of the single ring of Fig. 3c and of the double ring configuration of Fig. 3d. The moment of inertia of objects sharing an axis of rotation are additive if the moment of inertia of the double ring is two times the moment of inertia of the single ring. According to your results, are the moments of inertia of objects sharing an axis of rotation additive? To substantiate your answer, determine the percentage difference between the moment of inertia of the double ring and twice the moment of inertia of the single ring.

4. The analysis of this interpretation should show that the moment of inertia increases as the distribution of mass moves farther away from the axis of rotation. You will compare the moments of inertia of the rings of configurations of Figures 3c and 3f, the rings of configurations of Figures 3c and 3e.

   (a) Use $I_{ring} = I_{tot} - I_{s+b}$ and your results of interpretation 1 to determine the moment of inertia of the single ring of Fig. 3f. The moment of inertia of the ring of Fig. 3c has been determined in interpretation 2. The mass distribution of the ring of Fig. 3c is farther from the axis of rotation than that of the ring of Fig. 3f, is the moment of inertia of the ring of Fig. 3c greater than the moment of inertia of the ring of Fig. 3f?

(b) We have shown in interpretation 2 that the moments of inertia of objects sharing an axis of rotation are additive. This implies that the moment of inertia of one of the rings of Fig. 3e is half of that of the two rings, thus, use $I_{ring} = \frac{1}{2}\left(I_{tot} - I_{s+b}\right)$ to determine the moment of inertia of one of the rings of Fig. 3e. The mass distribution of one ring of Fig. 3e is farther from the axis of rotation than that of the ring of Fig. 3c, is the moment of inertia of one ring of Fig. 3e greater than the moment of inertia of Fig. 3c?

According to your previous two comparisons, how does the moment of inertia change as the distribution of mass moves farther away from the axis of rotation?

5. Use the theoretical equations for the moments of inertia of Figure 2 to determine the theoretical moments of configurations of Figure 3b, 3c, 3e, and 3f. The mass $M$, radii $R_1$ and $R_2$ are taken from Procedure 2. You can average the mass and the two diameters of the two rings. Compare these theoretical results with the correponding experimental moments of inertia previously determined in interpretation 2, 3, and 4. Generate a table summarizing the configuration, the experimental and theoretical moments of inertia and percentage difference.

6. The moment of inertia of a cylindrical ring with respect to its longitudinal axis is $I = \frac{1}{2}M({R_1}^2 + {R_2}^2)$. Use the parallel axes theorem to show algebraically and theoretically (no numbers allowed, just variables) that the moment of inertia of a cylindrical ring with respect to an axis of rotation touching the outer edge of the ring and parallel to the longitudinal axis is $I = \frac{1}{2}M({R_1}^2 + 3\,{R_2}^2)$. Calculate $I = \frac{1}{2}M({R_1}^2 + 3\,{R_2}^2)$, does the experimental moment of inertia of the two rings of Fig. 3e divided by two confirm this theoretical result?

7. (exploratory question, there is no wrong answer) The y-intercepts of your linear regression of interpretation 1 represents the frictional torque for that particular configuration. Generate a table of values with columns of configuration, frictional torque, total moment of inertia, total mass of the configuration. Does the frictional torque change much for different configurations? If it does, do the total moment of inertia and mass associated with the configuration affect the frictional torque? If so, is the relationship

proportional or inversely proportional? Plot a graph of frictional torque vs. $I_{tot}$ and mass $M$ to substantiate your hypothesis.