Newton's Second Law: An Introduction

1 Purpose

To verify Newton's 2^{nd} Law by showing that the acceleration of a body is proportional to the net external force on that body and that the acceleration is inversely proportional to the accelerated mass. To determine the force of friction between an object and surface and to show that the coefficient of static friction in greater than that of kinetic friction.

2 Theory

2.1 Newton's Second Law

Newton's Second Law states that the vector sum of the external forces on a body is equal to the mass times the acceleration of the body.

$$\sum_i \vec{F}_i = m\vec{a} \tag{1}$$

where $\vec{F}_i$ are the external forces on the body, m is the mass and $\vec{a}$ is the acceleration of the body.

Applying Newton's 2^{nd} law to the following situation,

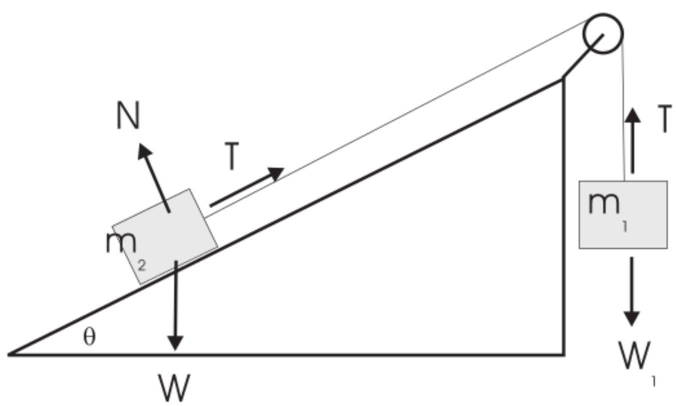

Figure 1: Two blocks and incline plane configuration

gives

$$a = \frac{m_1 - m_2 \sin\theta}{m_1 + m_2} g \tag{2}$$

for an horizontal surface, $\theta = 0$, which gives

$$a = \frac{m_1 g}{m_1 + m_2} \tag{3}$$

2.2 Friction

Attractive interactions between surfaces generate a force of friction opposite the direction of motion or against the applied force. The force of friction between two stationary objects in contact is the static friction and has a maximum of

$$f_{s,max} = \mu_s N \tag{4}$$

where $f_{s,max}$ is the maximum force of static friction, μ_s is the coefficient of static friction, and N is the normal force.

If the two objects in contact are moving relative to each other, the friction force is the kinetic friction and is given as

$$f_k = \mu_k N \tag{5}$$

where f_k is the force of kinetic friction, μ_k is the coefficient of kinetic friction. Typically the coefficient of static friction is greater than the coefficient of kinetic friction for two surfaces in contact.

3 Procedure

3.1 Newton's 2^{nd} law and force of kinetic friction

1. Assemble equipment as instructed by lab instructor.

2. Measure the mass of the cart on horizontal surface (m_2).

3. For an incline plane with an angle of inclination equal to zero $\theta = 0°$ (horizontal surface), obtain the acceleration of the system of the hanging mass $m_1 = 50g$ and cart mass $m_2 + 80g$ without friction. The acceleration without friction is obtained by the slope of a velocity vs. time graph in datastudio.

4. Transfer 20 g (2 x 10g weights) from the cart m_2 to the hanging mass m_1 in order to keep $m_1 + m_2 =$ constant. Repeat the previous procedure for a total of 5 different values of m_1 and m_2. Fill-in Table 1.

Table 1			
m_1(g)	m_2(g)	$m_1 + m_2$(g)	a (m/s^2)

5. Without friction, obtain the acceleration for 5 different m_2 but keeping $m_1 = 50g$ constant, fill-in Table 2. Start with an empty cart and increase successively by 20 g.

Table 2			
m_1(g)	m_2(g)	$m_1 + m_2$(g)	a (m/s^2)

3.2 Force of kinetic friction

6. Assemble equipment as instructed by lab instructor.

7. Measure the mass of the PVC block (m_{pvc}).

8. With friction between a block of PVC and a horizontal PVC surface, obtain a velocity versus time of the system of the PVC block of mass m_{pvc} and hanging mass m_1 where at one point the mass m_1 reaches the ground. The mass m_1 should be large enough to overcome the static friction. Note the position of m_{pvc} where m_1 reaches the ground. Use the Capstone software to determine the maximum velocity of m_2. This corresponds to the point where m_1 reaches the ground and has no effect on m_2. Note the distance that it takes for m_2 to stop. Fill-in the first row of Table 3.

9. Increase the mass m_2 by 20 g and repeat the previous procedure for a total of 5 different values of m_2. Fill-in Table 3.

Table 3	$x_i(m)$=	
m_{pvc}(g)	v_{max} (m/s)	Δx (m)

3.3 Force of static friction

10. Measure the mass of the PVC block (m_{pvc}).

11. Raise on end of the smaller PVC surface on a laboratory jack to create an angle of inclination. You can measure the angle of inclination using the protractor and plumb line. Place the small empty block of PVC on the inclined surface always at the same position. Slowly increase the angle of inclination until that the PVC block (m_{pvc}) starts to slide. Record this critical angle θ_c where the component of the block's weight overcomes the maximum static friction and the block starts to move.

12. Increase the mass of m_{pvc} by 50 g and repeat the previous procedure for a total of 5 different values of mass m_{pvc}. Fill-in Table 4.

Table 4					
m_{pvc} (g)					
(θ_c) (deg)					

4 Interpretation of Results

1. Plot a graph of a versus $F_{net} = m_1 g$ using the without friction values of Table 1. Use a value of g to be 9.785 N/kg. Convert your gram units to kilogram. All variables are scalar quantities. Use linear regression and find the best fit slope. Compare (using the percentage difference formula) the best-fit slope to $1/(m_1 + m_2)$. How do you interpret this result, i.e. how does this show that $a \propto F_{net}$?

2. Plot a graph of a versus $1/(m_1 + m_2)$ using Table 2. Convert your gram units to kilogram. All variables are scalar quantities. Use linear regression and find the best fit slope. Compare (using the percentage difference formula) the best-fit slope to F_{net}. How do you interpret this result, i.e. how does this show that $a \propto \frac{1}{m_1+m_2}$?

3. The deceleration due to friction is calculated using the third kinematic equation and the maximum speed given on the velocity versus time graph in datastudio, and the distance to stop (Δx) in Table 3: $a_{kf} = \frac{(v_{max})^2}{2\Delta x}$. Calculate a_{kf}. Determine the force of kinetic friction generated between the two PVC surfaces using the values of Table 3, where $f_k = m_{pvc}\, a_{kf}$ and $N = m_{pvc}\, g$, plot a graph of f_k vs. N. Use a value of g to be 9.785 N/kg. All variables are scalar quantities. Use linear regression to obtain the slope which is equal to the coefficient of kinetic friction μ_k of the two PVC surfaces in contact.

4. Just before slipping the net force on the PVC block is $\vec{F}_{net} = m_{pvc}\, g\ \sin\theta_c - \mu_s\, m_{pvc}\, g\ \cos\theta_c = 0$. This simplifies to $\mu_s = \tan\theta_c$. Calculate the coefficient of static friction for each critical angle and average your values.

   Do your results confirm that the coefficient of static friction is graeter than the coefficient of kinetic friction?

   Compare (% difference) your coefficients of static and kinetic friction values to those cited in the following reference.

   `https://www.hs-plasticgroup.com/new_detail/nid/73081.html`