Projectile Motion

1 Purpose

To show that a given projectile position in its trajectory can be obtained with two different firing angles and to show that for a given angle the trajectory of a projectile is parabolic.

2 Theory

The trajectory of a projectile where $x_0 = 0$ and $y_0 = 0$ is given by

$$y = (\tan\theta)x - \left(\frac{1}{2}\frac{|a_g|}{{v_0}^2\cos^2(\theta)}\right)x^2 \tag{1}$$

This equation is quadratic in x which gives the shape of a parabola.

The range of a projectile is affected by four parameters: the initial instantaneous speed and initial direction (angle with respect to the horizontal) of the projectile, the initial height, and the acceleration due to gravity. The acceleration due to gravity in the lab is assumed to be $|a_g| = 9.785$ m/s^2.

For an arbitrary initial direction θ at initial projectile height $h = 0$, the horizontal range is

$$R = \left(\frac{{v_0}^2}{a_g}\right)\sin(2\theta) \tag{2}$$

3 Procedure

1. Setup the equipment as instructed by your professor.
2. Adjust the time-of-flight pad such that the horizontal distance between it and the launcher is exactly 1.000 m. For an horizontal firing angle of zero degrees (full compression), point the projectile launcher towards the time-of-flight sensor, making sure that the photogate LED at the exit point of the launcher is OFF, fire the projectile. Determine the time of flight of your projectile using the capstone software.
3. Repeat 5 times. Expected time of flights for a distance of 1.000 m is between 0.14 and 0.2 s.
4. Position a small container with opening oriented diagonally towards the launcher approximately 2 to 3 meters from the launcher at a height between 0.5 to 1.5 meters.
5. Determine the low trajectory firing angle to get the projectile into the container.

6. Position a small ring at the top most part of the trajectory such that the projectile moves through the ring on its way to the container. Use a video of the projectile motion through the ring to make that the ring is at the top most part of the trajectory. If the target is at the same height as the initial projectile position, the apex of the trajectory is halfway between both positions. If the target is higher than the initial position, the apex moves closer to the target. If the target is lower than the initial position, the apex moves closer to the projectile launcher.

7. Use a plumb bob and rulers to measure the geometry of the system as indicated in the following figure. Try to achieve a precision to the millimeter, for example, $y_1 = 88.0\,cm = 0.880\,m$.

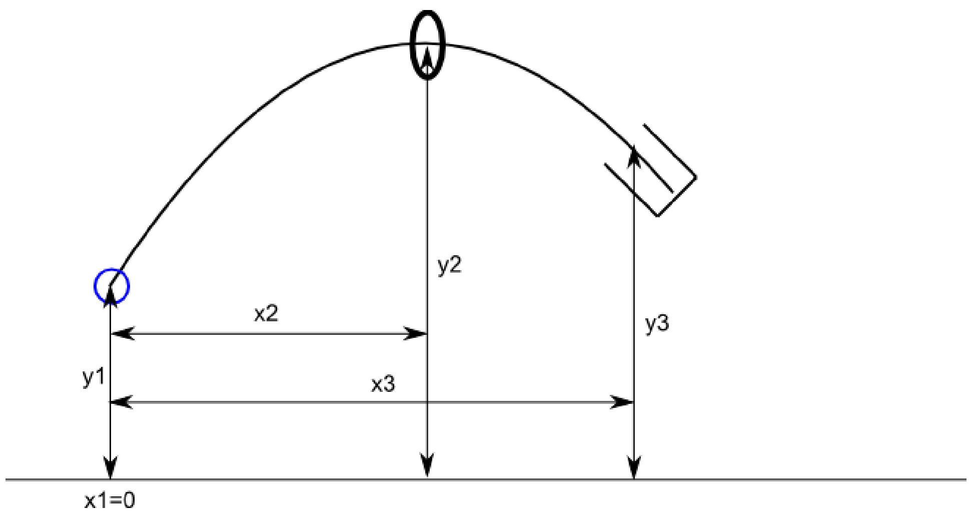

Figure 1: Geometry of the experimental setup

8. Determine the high trajectory firing angle to get the projectile into the container.

4 Interpretation of Results

1. Determine the initial speed v_0 of the projectile by dividing the horizontal distance $\Delta x = 1.000\,m$ to the time-of-flight sensor by the **average** time of flight, as in $v_0 = \frac{1.000\,m}{t_{avg}}$.

   Determine the y-component of the initial velocity, v_{0y}, for your low trajectory firing angle using $v_{0y} = v_0 \sin\theta_{low}$, and calculate the maximum vertical displacement Δy of the projectile in its trajectory. Use the third kinematic equation, ${v_y}^2 = {v_{0y}}^2 - 2|a_g|\Delta y$ where v_y is zero at the top of its trajectory.

   Compare your calculated vertical displacement to the experimental vertical displacement: $\Delta y = y_2 - y_1$ according to figure 1. Use the % difference formula to compare.

2. Determine the low and high firing angles grafically. The trajectory equation (Eq. 1) can be manipulated to obtain $2\ y\ ({v_0}^2)\cos^2(\theta) = x\ ({v_0}^2)\sin(2\theta) - |a_g|\ x^2$.

   Using the position of your container ($x = x_3$ and $y = y_3 - y_1$) and your value of v_0 of interpretation 1 and $a_g = 9.785\ m/s^2$, plot two curves, using continuous lines, on one graph: the left side and right side of the above trajectory equation function of θ.

   - In Excel, generate values from 20 to 80 incrementing by 1 in the first column.
   - In the second column calculate the left side of the above trajectory equation where the angles are given by the cells in the first column. In Excel the arguments of the trigonometric functions must be in radians.
   - In the third column, calculate the right side of the above equation.
   - Plot the 2nd and 3rd column from 20.0° to 80.0° at every 1.0°. The curves should intersect at two points which correspond to your high and low firing angles. Provide a copy of the graph and the two rows in Excel where the intersection occurs.
   - Compare using the % difference formula your experimentally and graphically determined firing angles.

3. The trajectory equation can be re-written as:

$$y_3 - y_1 = (\tan\theta)x_3 - \left(\frac{0.5|a_g|}{v_0^2}[1+\tan^2(\theta)]\right){x_3}^2$$

   Making a variable substitution, such as, $z = \tan\theta$ and $k = \frac{0.5|a_g|}{v_0^2}$, distributing, and solving using the quadratic formula gives:

$$\theta_1 = \tan^{-1}\left(\frac{x}{2kx^2} - \frac{\sqrt{x^2 - 4kx^2(y+kx^2)}}{2kx^2}\right)$$

   and

$$\theta_2 = \tan^{-1}\left(\frac{x}{2kx^2} + \frac{\sqrt{x^2 - 4kx^2(y+kx^2)}}{2kx^2}\right)$$

   - Using your container position, $x = x_3$, $y = y_3 - y_1$, v_0, a_g, $k = \frac{0.5|a_g|}{v_0^2}$ and the above two equations, calculate your theoretical low and high firing angles. .

4. Using your experimentally determined low and high angles θ_{low} and θ_{high}, v_0, a_g, and y_1, and the equation below:

- In Excel, using the trajectory equation (Eq. 1) but **add** the y_1 variable to the right side of the equation: $y = (\tan\theta)x - \left(\frac{1}{2}\frac{|a_g|}{{v_0}^2\cos^2(\theta)}\right)x^2 + y_1$, plot the two trajectories of your projectile from $x = 0$ m to 4 meters at every 0.1 m. Use a continuous line plot without data points.
- Indicate the experimental position of the ring and container on the trajectory using a second set of data points and x-y scatter on the same graph.
- Do your trajectories converge on the container position? Does the the apex of the low angle trajectory correspong with the ring position?