Position, Velocity, and Acceleration

1 Purpose

To use the kinematic equations to determine when and where a collision will ocur between two objects moving in a straight line up and down an incline plane. This result will also be verified experimentally.

2 Theory

The following kinematic equations may be used for the experiment.

$$\vec{x}_f = \vec{x}_i + \vec{v}_i \Delta t + \frac{1}{2}\vec{a}(\Delta t)^2 \tag{1}$$

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t \tag{2}$$

$$v_f^2 = v_i^2 + 2\vec{a}\cdot\Delta\vec{x} \tag{3}$$

3 Procedure

1. Set-up the equipment and sensors as indicated by your instructor.

2. Measure the angle of inclination of the air track using your smart phone.

3. Record the initial position of the up (x_{1i}) and down-going (x_{2i}) carts (leading edge) and the position of the photogate following the smart-pulley (x_{sp}) with the leading edge of the up-going cart. Also record the positions of the three photogates ($x_{pg1}, x_{pg2}, x_{pg3}$) used to calculate the acceleration of the carts on the incline plane.

4. For a given hanging mass and without the down-going cart, propel the up-going cart and take a smartphone video to determine the position the leading edge of the cart at its highest point where its velocity is momentarily zero. Take a screencapture of the capstone desktop window. Repeat for a total of three times.

5. Perform the experiment with the up and down-going cart and obtain a movie of the collision between two carts using your smart phone.

6. From the capstone desktop of the computer closest to the bottom of the incline plane export the smart-pulley velocity vs. time data. Perform a screen capture of the capstone desktop and perform a screen capture of the capstone desktop of the computer closest to the top of the incline plane.

4 Interpretation of Results

1. Use your velocity-time data of procedure 5 to plot a graph of v versus t in Excel. Use linear regression to obtain the initial acceleration of the up-going cart when it is being pulled by a falling mass.

   Determine the final velocity of the up-going cart as it is pulled by the hanging mass and passes underneath the smart-pulley using the third kinematic equation. It should be similar to the maximum v_f value reported by the capstone desktop. Find the percentage difference between the two values. The percentage difference is given by:

$$\% \text{ difference} = \frac{|A - B|}{\frac{A+B}{2}} \times 100 \tag{4}$$

2. Use the third kinematic equation and the average of the maximum velocity given by the screenshots and the average displacement of the up-going cart between the highest point and the first photogate position of procedure 4 to determine the magnitude of the experimental acceleration of the up-going cart when freely moving on the frictionless incline plane.

   Determine the magnitude of the experimental acceleration of the down-going cart using the three photogates' positions and time intervals. Δt_{12} and Δx_{12} are the time interval and the magnitude of the displacement between the first and second photogates. Δt_{13} and Δx_{13} are the time interval and magnitude of the displacement between the first and third photogates. It can be shown using the first kinematic equation applied to the first and second photogate and to the first and third photogate that the magnitude of the acceleration of the down-going cart is

$$a_{exp} = \frac{\frac{\Delta x_{13}}{\Delta t_{13}} - \frac{\Delta x_{12}}{\Delta t_{12}}}{0.5(\Delta t_{13} - \Delta t_{12})}$$

3. The acceleration of the up- and down-going cart should be the same, so average the magnitude of the two experimental values to obtain the average acceleration of the cart. The theoretical value for the cart acceleration magnitude on the incline plane is given by $a = a_g \sin\theta$ where a_g is the acceleration due to gravity and is equal to $9.785\ m/s^2$ for our geographical location and θ is the angle of inclination of the air track. Calculate the theoretical acceleration and compare it with your experimental value using the percentage difference formula.

4. Consider the following kinematic situation:

   The up-going cart at t=0 has an initial position given by the position of the first photogate and initial velocity given by the maximum velocity of the capstone desktop of procedure 5 and an experimental acceleration given by interpretation 6. The down-going cart has at t=0 an initial position towards the upper end of the air track with zero initial velocity and an experimental acceleration given by interpretation 6. Be careful with the signs of your kinematic variables, they are vectors. The acceleration vector is negative. Objects moving up (to the right) of the air track are positive and those moving down are negative.

(a) Use the first kinematic equation applied to each cart to determine at what time and position do the carts collide. Hint: the final positions of both carts are the same at collision.

(b) Use the percentage difference equation to compare your calculated collision position using kinematics to the experimental collision position from your video.